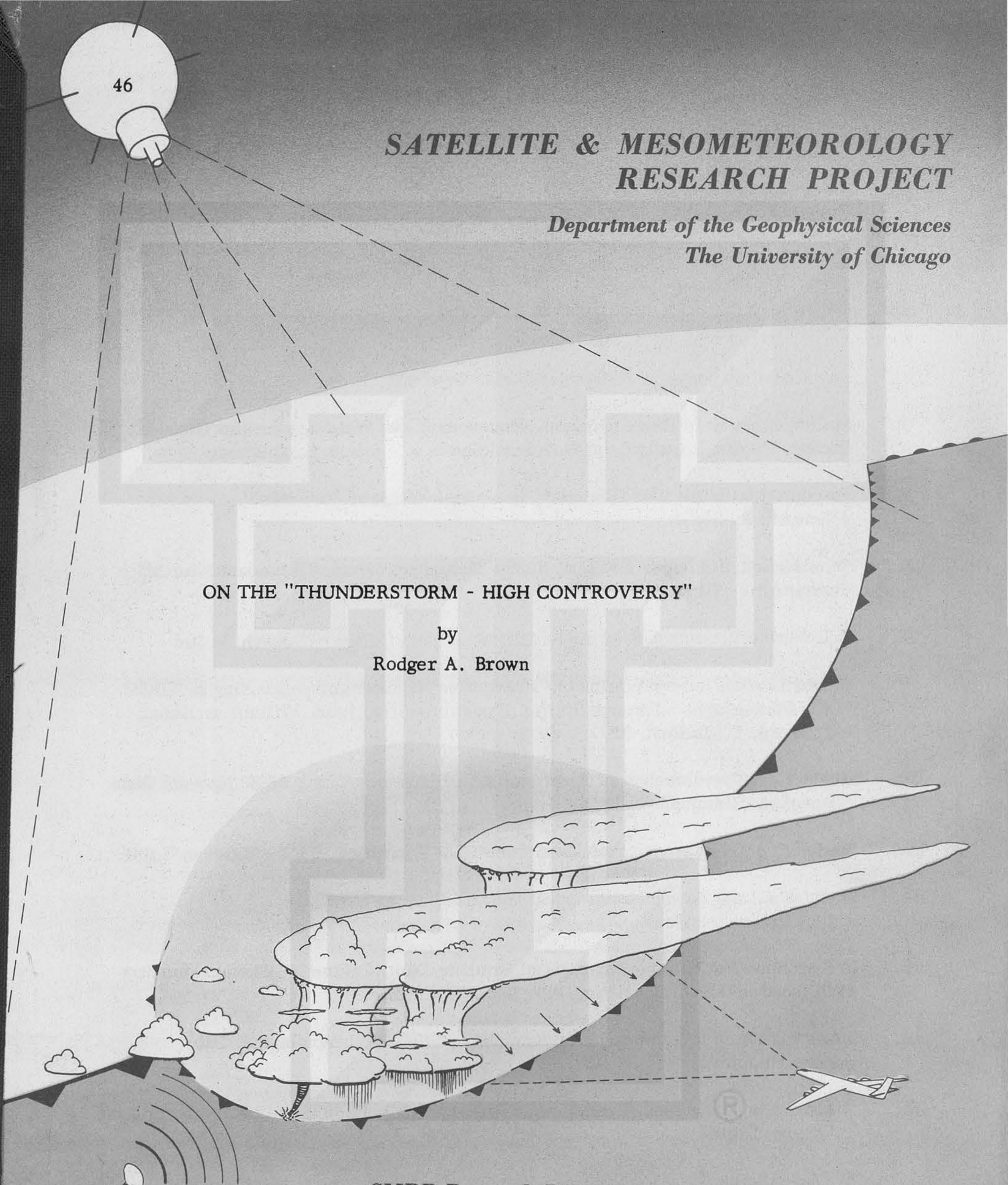

SATELLITE & MESOMETEOROLOGY RESEARCH PROJECT

Department of the Geophysical Sciences
The University of Chicago

ON THE "THUNDERSTORM - HIGH CONTROVERSY"

by

Rodger A. Brown

SMRP Research Paper

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ON THE "THUNDERSTORM - HIGH CONTROVERSY"

Rodger A. Brown

Department of the Geophysical Sciences

The University of Chicago

Chicago, Illinois

ABSTRACT

From 1942 to 1952, what is referred to as the "Thunderstorm - High Controversy" took place among meteorologists throughout the world. The controversy came about when attempts were made to explain, from dynamical considerations (i.e., vertically accelerating and decelerating air), the high pressure in the pressure nose which forms beneath thunderstorms. Some meteorologists attributed the pressure rise to upward accelerating air, but others concluded that such accelerations should produce a pressure dip. Authors, who refer to the papers involved, mistakenly assume that precipitation downdrafts were being considered as the cause of the rise, as is now accepted. Therefore the purpose of this paper is twofold: first, to investigate the assumptions made in the papers and point out implicit and explicit errors; and, second, to explain pressure changes beneath an idealized developing thunderstorm in terms of the integrated influences of hydrostatic and hydrodynamic pressure within and beneath the cloud.

1. Introduction

In that phase of meteorology which has come to be known as mesometeorology, one of the early problems of interest was to explain the area of higher pressure that is found beneath thunderstorms. One of the earliest explanations was put forth by Abercromby (1875). He took readings from an aneroid barometer at five-minute intervals as a thunderstorm approached his location. It was noted that as the clouds, which had been growing rapidly along the leading edge, passed overhead, there was a rapid rise in pressure. Paying no attention to rain, which had started to fall in the middle of the rapid rise, Abercromby theorized that the high pressure must be due to the dynamic effects of the strong updrafts within the growing clouds.

Of many later papers that have touched upon reasons for the high pressure,

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Humphreys' (1914) is one of the most outstanding, especially for his time. To quote part of his discussion:

> Before the onset of a thunderstorm there usually if not always is a distinct fall in the barometer . . . Just as the storm breaks, however, the pressure rises very rapidly, almost abruptly, usually from 1 to 2 millimeters, fluctuates irregularly, and finally as the storm passes again becomes rather steady but at a somewhat higher pressure than prevailed before the storm began.
>
> The cause of the pressure changes is, doubtless, rather complex. The decrease in the absolute humidity and the decrease in temperature both tend to increase the atmospheric pressure, and, presumably, each contributes its share. Both these effects, however, are comparatively permanent, and while they may be mainly responsible for the increase of pressure that persists after the storm has gone by, they probably are not the chief factors in the production of the initial and quickly produced pressure maximum. Here at least two factors, one obvious, the other inconspicuous, are involved. These are: a. the rapid downrush of air, and b. the interference to horizontal flow caused by the vertical circulation.

The long-lasting pressure rise that Humphreys referred to is now called the pressure dome and the initial, short-lived pressure maximum, which is superimposed upon the dome, is called the pressure nose or thunderstorm nose or Gewitternase or crochet d'orage.

Sawyer (1946) has paraphrased part of Humphreys' reasoning as follows:

> Heavy rain associated with cumulonimbus cloud cools the air through which it falls by evaporating into it. This increases the density of the air column and it contracts; pressure thus falls at the top of the cooled column and air flows into the new low-pressure centre at that level. This results in a rise of pressure at the ground. Were it not for friction in the layers near the surface, the air flowing out from the rising pressure at the ground would soon balance that flowing in at higher levels, but as the outflow is retarded near the ground more air enters than leaves the air column and surface pressure continues to rise.

It appears, therefore, that the pressure dome is due to static pressure effects caused by the additional weight of dense cold air in the subcloud region--the coolness being attributed to evaporational cooling of the rain. On the other hand, the pressure nose is caused primarily by the static effect of a descending mass of liquid and/or solid water and by the dynamic effects of accelerations and decelerations in the downdraft and of the retardation of the outflowing air by surface friction. (For an outline of additional factors that are partial contributors to the formation of the pressure dome and nose, the reader is referred to U.S. Weather Bureau, 1949).

The first to give any detailed discussion of the hydrodynamical pressure contributions to the total pressure change beneath a thunderstorm was Levine (1942). His major contribution to the Thunderstorm-High Controversy was to focus attention on the effects that vertical accelerations have on surface pressure; however, it is unfortunate that he made the erroneous assumption that the high pressure is due mainly to upward accelerations within the cumulonimbus. None of the controversy that his paper evoked during the following decade recognized the fallacy of that basic assumption; the main point of disagreement was whether dynamic pressure is positive or negative on the ground beneath an updraft. An even more confusing fact is that authors, who refer to Levine's paper and papers of other participants in the controversy, are under the misconception that the papers deal with accelerations within precipitation downdrafts (see, e.g., Kaplan, 1943; Bleeker and Andre, 1950; Fujita, 1959).

The purpose of this paper is twofold: 1) to discuss the general features of the distribution of hydrostatic and hydrodynamic pressure within an idealized thunderstorm and then to show how cloud dynamics are controlled by the mutually acting horizontal total pressure and vertical hydrodynamic pressure gradients, and 2) to outline the controversy (which deals with trying to explain the symmetrical pressure nose) using the above-mentioned discussion as a reference source.

2. Distribution of Hydrostatic and Hydrodynamic Pressure in Thunderstorms

The joint role of hydrostatic and hydrodynamic pressure distributions on the dynamics of convective clouds have never been adequately investigated. In this section it will be possible to obtain realistic pressure distributions for a cumulus congestus and cumulonimbus by using simplified vertical distributions of density and vertical velocity. The clouds are assumed to be axial symmetric, stationary, nonrotating, and developing in an environment with no wind shear. Environmental values of density are for the U.S. Standard Atmosphere (U.S. Committee Standard Atmosphere, 1962).

To begin with, total pressure (p) can be defined as

$$p = p_s + p_d , \tag{1}$$

where p_s is hydrostatic pressure and p_d is hydrodynamic pressure. Instead of dealing with an absolute value of pressure at a given level in a cloud, it is more convenient to consider a <u>differential pressure</u> (Δp) which can be defined as the pressure at a given point within the cloud relative to the pressure outside the cloud at the same level; differential pressure can be positive or negative. Equation (1) can be expressed in terms of differential pressure in the following way:

$$\Delta p = \Delta p_s + \Delta p_d \tag{2}$$

Pressure at a particular elevation is a reflection, or summation, of events that take place at all levels from the top of the atmosphere (∞) down to that elevation (z). To express this mathematically by making use of the vertical equation of motion, we have

$$\begin{aligned} p(z) - p(\infty) &= \int_{\infty}^{z} \frac{\partial p}{\partial z}\, dz = -\int_{\infty}^{z} \rho \left(g + \frac{dw}{dt} \right) dz \\ &= -\int_{\infty}^{z} \rho \left(g + \frac{\partial w}{\partial t} + v_r \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) dz , \end{aligned}$$

where g is acceleration due to gravity, w is vertical velocity, v_r is radial velocity, ρ is density, and r and z are in radial and vertical directions, respectively. The following assumptions are implicit in the above equation: viscous forces are negligible (at least one or two orders of magnitude smaller than other terms); influence of Coriolis force is negligible (true for scale of motions found in thunderstorms); and there is no rotation (a dubious assumption for most thunderstorms). The equation can be put into the following form

$$p(z) - p(\infty) = -\int_{\infty}^{z} \rho\left(g + v_r \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z}\right) dz , \qquad (3)$$

if one further assumes that the vertical velocity is in a steady-state condition (far from true in a growing cloud, but assumed to be appropriate in the two stages of development used below).

By making use of the differential pressure concept, (3) can be expanded into

$$\Delta p(z) - \Delta p(\infty) = -\int_{\infty}^{z} \left\{ \rho\left[\Delta g + \Delta\left(v_r \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z}\right)\right] + \left[\overline{g + v_r \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z}}\right] \Delta\rho \right\} dz , \qquad (4a)$$

where it is to be remembered that Δ represents a horizontal difference; the horizontal bar signifies a horizontal mean between cloud and environment. By taking into account that g does not vary horizontally and by splitting the integration into two stages--top of atmosphere (∞) to top of cloud (z_T) and top of cloud to level z --the equation becomes

$$\left[\Delta p(z) - \Delta p(z_T)\right] + \left[\Delta p(z_T) - \Delta p(\infty)\right] = -\int_{z_T}^{z} B\, dz + \int_{\infty}^{z_T} B\, dz , \qquad (4b)$$

where

$$B = g\,\Delta\rho + \overline{\rho}\,\Delta\left(v_r \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z}\right) + \left(\overline{v_r \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z}}\right)\Delta\rho .$$

While it could be argued that there are descending motions above a cumulonimbus due to viscous and continuity considerations, they are negligible compared with vertical motions within the cloud. Therefore it can be assumed that there is no pressure difference between air above a cloud and air at the same level above the cloud's environment; this applies down to the very top of the cloud.

Again, due to continuity considerations, one would expect there to be descending air around a thunderstorm. That this is the case can be verified using photographs from meteorological satellites (see, e.g., TIROS VII, Orbit 686, near Gainesville, Florida about 1940 GCT on 4 August 1963) as well as in other ways. The cited TIROS photograph shows an isolated cumulonimbus surrounded by a marked cloud-free region which is three to four times larger than the diameter of the anvil; it is assumed that subsidence is responsible for the clear area. If the active updraft region in the cloud has a diameter on order of magnitude smaller than the diameter of the cloud-free area, the descending motion will be two orders of magnitude smaller than the average updraft velocity. Therefore, since environmental motions can be ignored and assumed zero,

$$\Delta\left(v_r\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=\left(v_r\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)_{\text{cloud}} .$$

Referring to Eq. (4b), the mean value of the radial and vertical velocity term in B is no larger than the above term for the cloud and $\Delta\rho$ is at least two orders of magnitude smaller than ρ ; therefore the right-most term in B is negligible compared to the other terms.

Taking the above considerations into account, (4) can be written as

$$\Delta p(z) = \Delta p_s(z) + \Delta p_d(z) , \qquad (5)$$

where

$$\Delta p_s(z) = -\int_{z_T}^{z} g\,\Delta\rho\,dz \qquad (6)$$

and

$$\Delta p_d(z) = p_d(z) = -\int_{z_T}^{z}\rho\left(v_r\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)dz . \qquad (7)$$

In (7), p_d and the integrand represent conditions solely in the cloud.

Computation of differential pressures. Equations (5)-(7) will be evaluated using simplified $\Delta\rho$ and velocity distributions for two model clouds representing different stages of development. The two cloud types are 1) cumulus congestus which is all updraft and 2) cumulonimbus which has developed a downdraft region and a surface mesosystem. Figures 1 and 2 show assumed vertical distributions of cloud density relative to environment and vertical velocity within cumulus congestus and cumulonimbus, respectively. As indicated in the figures, distributions are provided along the center of the cloud, midway between the center and edge, and along the edge of the cloud; in all cases it is assumed that values along the edge are the same as for the environment. All of the distributions, except those for the downdraft, are based on experience using rawinsonde observations to compute such parameters. Downdraft values within the cloud were assumed to be linear and their slopes were adjusted until they produced "realistic" results. The zero vertical motion and positive $\Delta\rho$ beneath cloud base in Fig. 2 reflect the presence of a surface mesosystem.

The model clouds are envisioned to extend from 2 km to 15 km; the top of the cumulus congestus has just reached 15 km. It is assumed that the high vertical velocities within the center of the congestus have been replaced by a precipitation downdraft in the cumulonimbus stage; it is much more realistic for the downdraft to form to one side of the central updraft, but the central downdraft will be used in order to take advantage of axial symmetry. The higher-than-average values for vertical cloud extent and vertical velocities are employed to produce more pronounced results.

Due to the axial symmetry of the clouds, the $\partial w/\partial r$ terms are zero along the center of the cloud. For the computations of the radial velocity term midway between the center and edge of the cloud, a simplified approach will be used. From the equation of continuity

$$\frac{v_r\,(r=R) - v_r\,(r=0)}{R-0} = -\frac{\partial w}{\partial z}\,(r=\tfrac{1}{2}R)\ ,$$

where R is the radius of the cloud. Since the radial velocity at the center of the cloud is zero and the horizontal mean of the radial velocity is approximately equal to the value at R in most cases, it follows that

$$\overline{v_r} \approx -R\frac{\partial w}{\partial z}\,(r=\tfrac{1}{2}R)\ .$$

The radial velocity term in (7) is then

$$v_r\frac{\partial w}{\partial r} = \overline{v_r}\,\frac{w\,(r=R) - w\,(r=0)}{R-0} = -\overline{v_r}\,\frac{w\,(r=0)}{R}$$

$$v_r\frac{\partial w}{\partial r} \approx w\,(r=0)\,\frac{\partial w}{\partial z}\,(r=\tfrac{1}{2}R)\ .$$

Therefore the radial velocity term at the midway point can be approximated by the product of the vertical velocity at the center of the cloud and the vertical shear of the vertical velocity at the midway point. As it turns out in the calculations, the inclusion of the radial velocity term makes the hydrodynamic pressure three times larger than it would have been had radial velocity been considered negligible.

Using the indicated distributions, Eqs. (6) and (7) were evaluated graphically; a planimeter was employed to integrate in 1-km steps from cloud top (where differential pressure is zero) to the ground. Figure 3 shows the resulting differential hydrostatic and hydrodynamic pressure distributions within a cumulus congestus and cumulonimbus. The left half of each cloud reveals the hydrostatic distribution and the right side, the hydrodynamic. The isobars, at 0.4-mb intervals, are skewed outward near the top of the cumulonimbus in order to take into account the diverging nature of the flow there.

Interpretation of hydrodynamic and total pressure gradients. Since two-or-three-dimensional (for axial symmetry) distributions of both components of pressure in clouds have not been determined before (from the author's knowledge), it would be wise to discuss the implications of considering both hydrostatic and hydrodynamic pressure. In ordinary meteorological discussions, one is not concerned with hydrodynamic pressure, but instead one considers pressure simply as pressure; surface air flows from high to low pressure and due to mass continuity considerations, air rises above the low. One gets into trouble, however, if he tries to explain vertical motion within a cloud from a total pressure gradient point of view, for air would have to flow from low differential pressure at the surface to high differential pressure up in the cloud.

Vertical accelerations are due solely to the vertical gradient of the hydrodynamic pressure. Even though mass continuity arguments are physically sound for determining the amount of vertical motion, they do not contain an explicit dynamical basis for the motion; the dynamical basis is, of course, the vertical gradient of the hydrodynamic pressure.

The arrows in Fig. 3 represent the directions of the primary pressure gradient forces that come into play in the dynamics of developing thunderstorms. The arrows for total pressure gradients are superimposed upon the hydrostatic pressure field. Since hydrostatic pressure is the predominant pressure component, the hydrostatic and total pressure gradients act in the same direction; a check of Fig. 3 reveals that the differential total pressure is about one-half of the differential hydrostatic pressure. For cumulus congestus, the total pressure gradient causes air to converge beneath the cloud and into the lower half of the cloud. The hydrodynamic gradient accelerates air upward to the level of maximum vertical velocity. At this level there is a total pressure gradient accelerating the air outward and a hydrodynamic gradient above the level which acts to rapidly decelerate the rising air. The net result is to have converging and upward moving air in the lower half of the cloud and diverging, rising air in the upper part.

For a rapidly rising cumulus cloud, the approximation that $\partial w / \partial t$ is negligibly small is no longer realistic; in fact, this term becomes one of the main factors that leads to the lowering of hydrodynamic pressure with height in the earlier stages of development. It seems reasonable that the level of maximum vertical velocity in a rapidly growing cloud should be closer to the top of the cloud than shown in Fig. 3a, leading to a shorter acting deceleration force. This, together with the outward pointing gradient force at the same level, would account for the "boiling" motion that is observed in the upper regions of actively growing clouds. (It becomes apparent from this discussion that the hydrodynamic pressure gradient force is due to vertical accelerations, and vice versa; a chicken-and-egg problem.)

When large drops within a thunderstorm first begin to descend, a very small pressure nose appears in the surface pressure; the nose is due primarily to the additional weight of the drops. As the number of drops in the downdraft increases and the draft extends down to the base of the cloud, the pressure nose increases due to combined hydrostatic and hydrodynamic effects. Once the downdraft has reached the ground, the nose is very pronounced and the diverging cool air leads to the formation of the pressure dome.

In the cumulonimbus, the pressure pattern is more complicated (Fig. 3b). One obvious feature is the doughnut-shaped torus (symmetric about vertical axis) of maximum differential hydrostatic and total pressure and minimum hydrodynamic pressure. Air descending through the hole in the torus accelerates toward the ground in the precipitation downdraft. This air, cooled by evaporation of raindrops, diverges outward at the surface from the region of high total pressure. In the subcloud and lower cloud regions, air continues to converge into the updraft which surrounds the idealized downdraft.

<u>Pressure beneath thunderstorm</u>. Figure 4 shows how the vertically integrated differential hydrostatic and hydrodynamic pressures contribute to the recorded (total) pressure traces on the ground. Beneath a cumulus congestus there is a pronounced low-pressure area. In the precipitation area beneath the cumulonimbus, the hydrostatic pressure is the main cause of the pressure nose (for a rapidly moving cloud, the nose usually forms near the leading edge of the dome). The pressure in the rest of the mesosystem exhibits nearly equal contributions from the cold outflowing surface air (hydrostatic) and from vertical accelerations within the cloud above it (hydrodynamic). In the figure, the horizontal extent of the pressure variations is assumed to be equal to the diameter of the main part of the cloud; due to converging surface air, the low pressure area ordinarily extends beyond the limit of the cloud.

It is possible to determine the net pressure rise due to the mesosystem by comparing the total pressure curve for the mesosystem with the total pressure curve for condit ons prior to the rain (dashes extension of low pressure area). The excess pressure (due to the presence of the mesosystem) is up to 3 mb in the pressure dome and just over 4 mb in the pressure nose. These values are in the range of what one would expect to find beneath an isolated cumulonimbus.

In this attempt to show the contributions of both the hydrostatic and hydrodynamic components to the overall pressure pattern in and beneath convective clouds, high vertical velocities were chosen for the calculations; in this way it was possible to show that high values of the magnitude of hydrodynamic pressure are only one-half to one-third of the magnitude of hydrostatic pressure in all regions within and below the cloud except the pressure dome part of the mesosystem. Within the pressure dome, the slightly greater hydrodynamic pressure is not a consequence of the mesosystem, but is due to vertical accelerations taking place in the cloud. Even though such factors as nonsteady motions and the net influence of high-and-low-level convergence and divergence fields have not been included in this model, it is believed that the pressure fields are realistic for the two stages of development that have been considered.

As an example of the hydrodynamic pressure that one would expect in the more complicated moving thunderstorm system which is common in regions having considerable vertical wind shear, Fujita (1963) used the thunderstorm model of Browning and Ludlam (1962) to compute the hydrodynamic component of the pressure. Figure 5a shows the two-dimensional stream lines and isotachs for vertical motion used by Fujita. A technique similar to that used in this paper was employed to compute the hydrodynamic pressure shown in Fig. 5b. The tilted updrafts and downdrafts produce a more complicated picture than that shown in Fig. 3b. It is noted that, while both have maximum negative pressure associated with maximum upward velocity and maximum positive pressure in the low-level updraft and diverging downdraft regions, the magnitude of the values are one-half to one order larger for the more violent type of storm which develops under conditions of pronounced vertical wind shear.

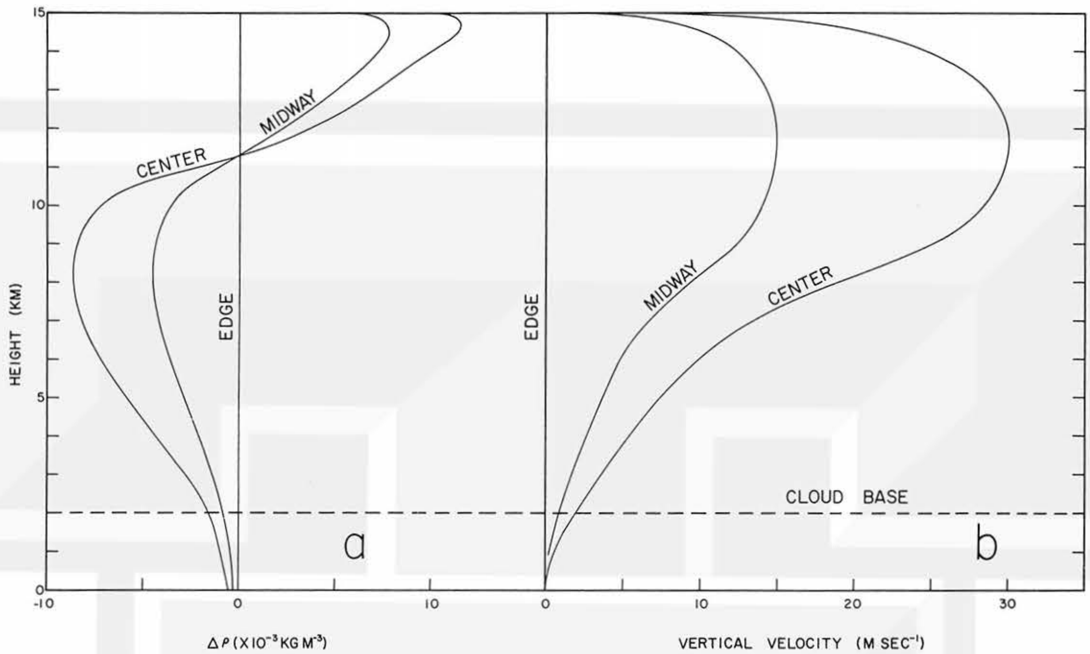

Fig. 1. Vertical distributions of a) density of air in cloud relative to that in environment at same level and b) vertical velocity within and below a well-developed cumulus congestus. The labeled curves are for center of cloud, region midway between center and edge, and edge of cloud; the edge of the cloud is assumed to have the same values as the environment.

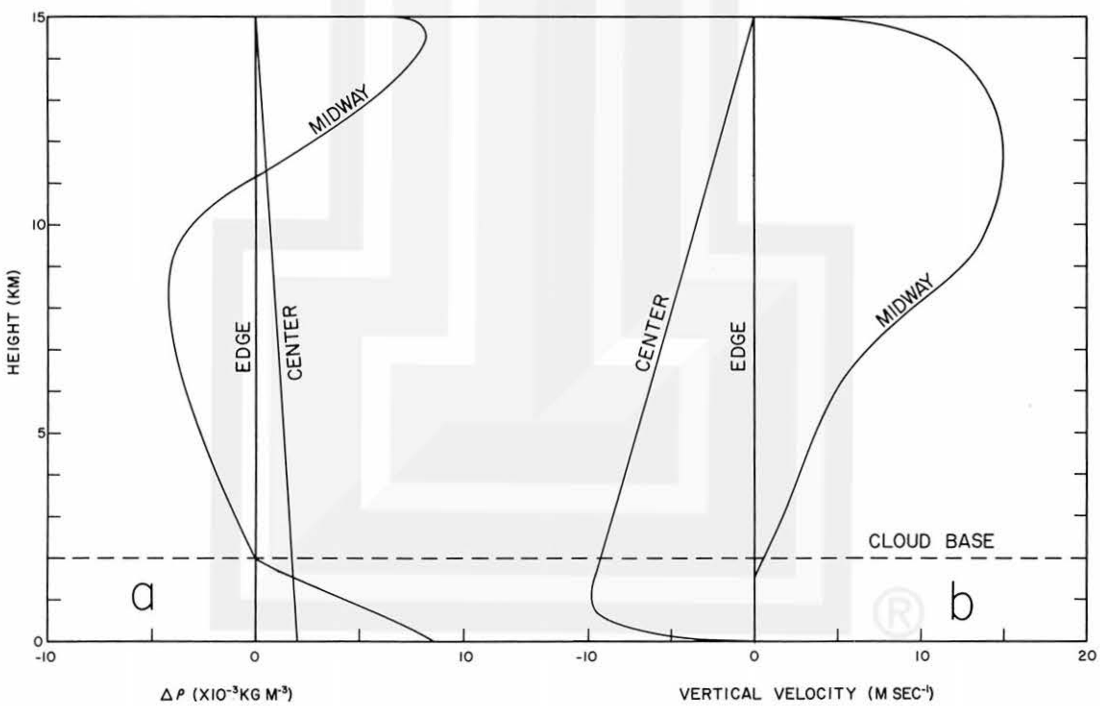

Fig. 2. Same as Fig. 1, but for a cumulonimbus.

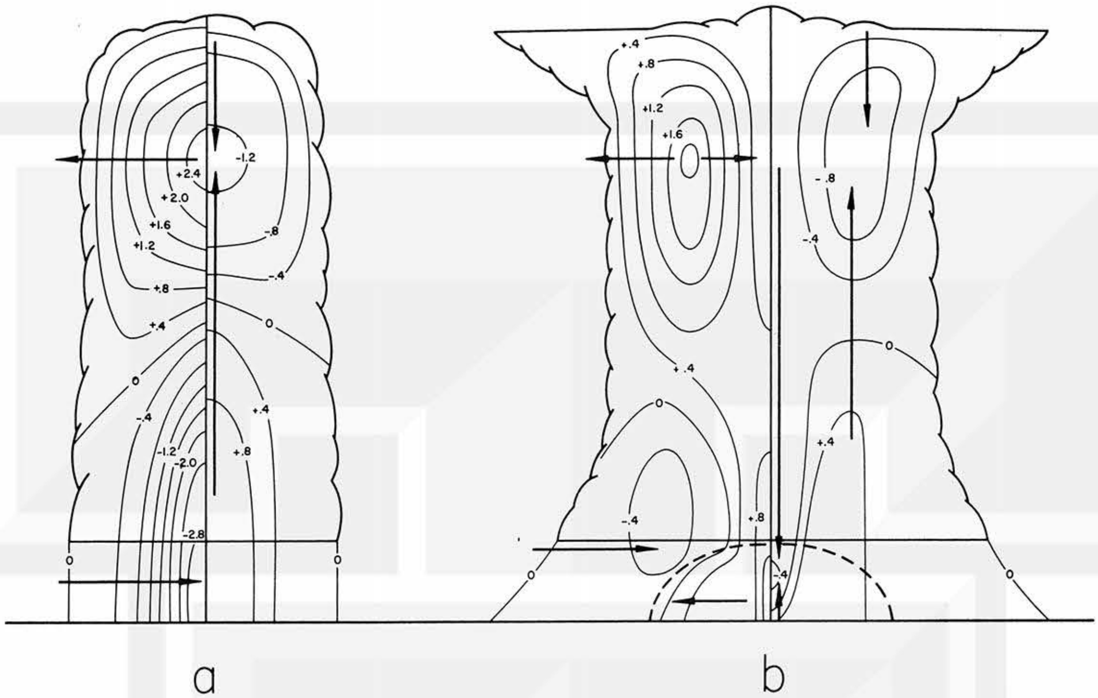

Fig. 3. Differential pressure distributions within and below a) cumulus congestus and b) cumulonimbus. Pressure is in millibars. The left half of each cloud shows differential hydrostatic pressure and the right side, differential hydrodynamic pressure. The arrows indicate the directions in which the main pressure gradient forces act; arrows for differential total pressure are superimposed on the differential hydrostatic pressure field. The dashed lines beneath the cumulonimbus outline a surface mesosystem.

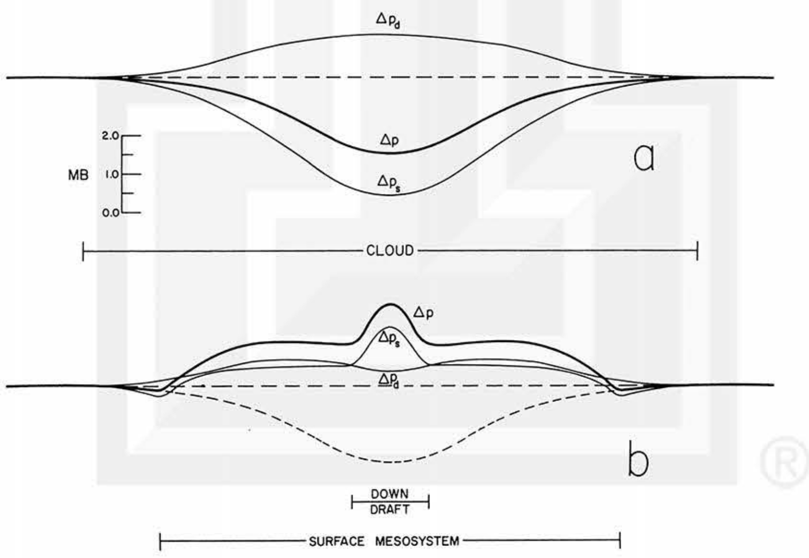

Fig. 4. Differential total (Δp), hydrostatic (Δp_s), and hydrodynamic (Δp_d) pressure distributions on the ground beneath a) cumulus congestus and b) cumulonimbus. The differential total pressure is the sum of the two components.

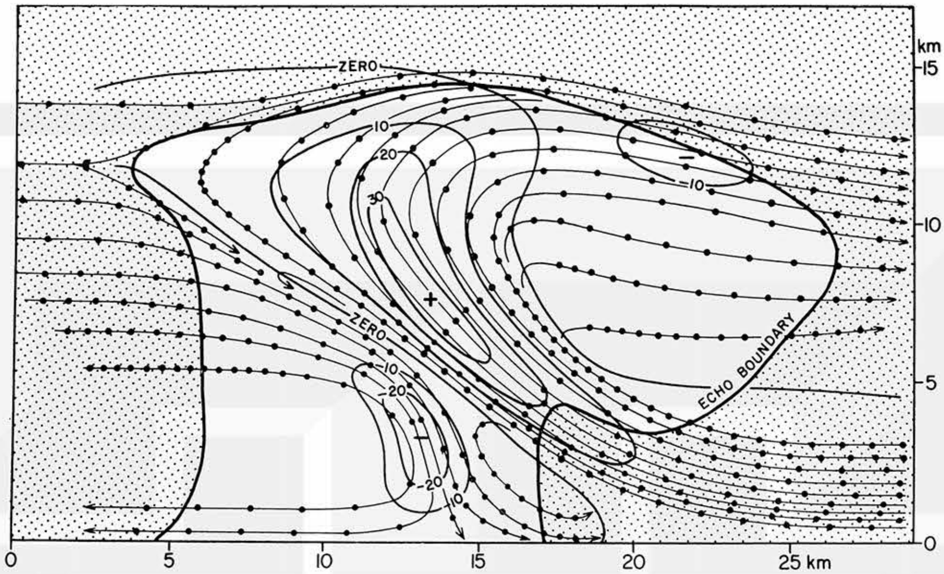

Fig. 5 a. Steady-state velocity data used by Fujita (1963) for computing hydrodynamic pressure from the Browning-Ludlam (1962) thunderstorm model. Isotachs of vertical velocity are drawn at 10 m sec^{-1} intervals. Black dots on the stream lines represent two-minute movements of air parcels.

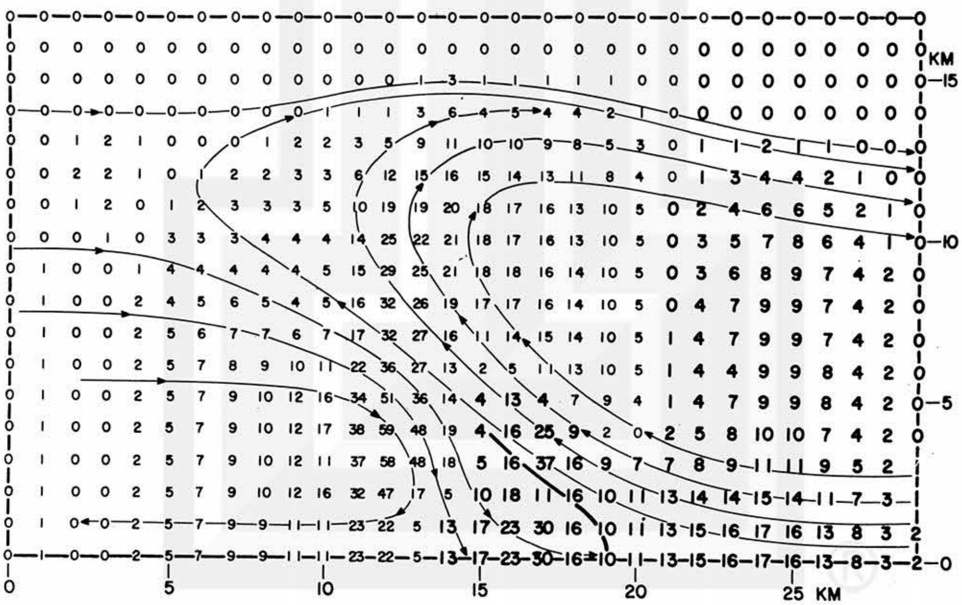

Fig. 5 b. Distribution of hydrodynamic pressure (units of 0.1 mb) for the same region shown in upper half of figure. Boldface numbers represent positive hydrodynamic pressure and lighter smaller ones, negative pressure. From Fujita (1963).

3. The International Controversy

In 1942 Levine, an American, presented a theoretical explanation for the formation of the symmetrical pressure nose that is found beneath thunderstorms. Referring to the work of Shaw and Dines (1905), he discounted the necessity for heavy rain to be present in order to produce the higher pressure because "they observed a marked pressure rise which occurred in connection with the formation of an extremely dark cloud over the station where only a few drops of rain fell". The obvious conclusion was: "Of all the suggested causes for the local pressure rise accompanying the thunderstorm, the dynamic one seems most likely."

By starting with the equation for vertical motion, Levine had apparently eliminated the hydrostatic part (using the hydrostatic relationship) and obtained something similar to hydrodynamic pressure on the ground as a function of the mean vertical acceleration from the ground to the level of maximum vertical velocity. Since the acceleration is zero at both of these levels, he considered only acceleration and not the effect of deceleration between level of maximum vertical velocity and cloud top.

Levine, as well as the other authors mentioned below, did attribute the cause of the pressure dome to both hydrostatic and hydrodynamic (for upward moving air) effects, but he felt that only the hydrodynamic effects of upward accelerating air could explain the sharper profile of the pressure nose. Based on the results in the last section, it is noted that Levine was entirely correct to say that vertical accelerations produce high pressure on the surface, but in order to interpret barograph traces, it is necessary to take into account the joint contributions of both hydrostatic and hydrodynamic effects. To attribute upward accelerations to the formation of the pressure nose does not agree with more recent findings (e.g. U. S. Weather Bureau, 1947; U. S. Weather Bureau, 1949) and the results shown in Fig. 4 indicate that it is the hydrostatic component due to falling rain that produces the nose in the total pressure trace.

Buell (1943a and b), also in the United States, expanded the scope of Levine's theoretical equations to include decelerations from the level of maximum vertical velocity to cloud top. Still attributing the thunderstorm pressure nose to upward accelerations, Buell found that the pressure on the ground was positive, in agreement with Levine's less adequate presentation.

However, Mal and Rao (1945) in India, using the approach of Levine and Buell, correctly showed that Buell had ignored an additional term in an expansion and therefore that dynamic pressure on the ground beneath an updraft is indeed negative. Since the computations of Levine and Buell, as well as those presented in Section 2, indicate that the dynamic pressure should be positive, it is advisable to outline the mathematics of the problem. (It must be kept in mind that Levine, Buell, and Mal and Rao all used the wrong physical explanation for the formation of the pressure nose.)

Mathematical model used by Levine, Buell, and Mal and Rao. For a chronological development, Levine's work will be mentioned first. If one ignores viscous dissipation in the vertical motion field and recognizes that the Coriolis force has a negligible effect upon motions within a cloud, the vertical equation of motion becomes

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = -g - w\frac{\partial w}{\partial z} \quad , \qquad (8)$$

where p is total pressure, g is acceleration due to gravity, ρ is density, and w is vertical velocity. The equation for the hydrostatic relation is

$$\frac{1}{\rho}\frac{\partial p_s}{\partial z} = -g \quad , \qquad (9)$$

where p_s is hydrostatic pressure. By combining (8) and (9) with the equation of state for an ideal gas, one obtains

$$\frac{1}{p}\frac{\partial p}{\partial z} - \frac{1}{p_s}\frac{\partial p_s}{dz} = -\frac{w}{RT}\frac{\partial w}{\partial z} \quad , \qquad (10)$$

where R is the gas constant for air and where T, the temperature, can be expressed as

$$T = T_0 - \gamma z \ ,$$

where T_0 is surface temperature and γ is temperature lapse rate. Integration of (10) from the ground to level z results in

$$\log\frac{p_0}{p} - \log\frac{p_{so}}{p_s} = \log\frac{p_0}{p_{so}} - \log\frac{p}{p_s}$$
$$= \int_0^z \frac{w}{R(T_0 - \gamma z)}\frac{\partial w}{\partial z}\,dz \ , \qquad (11)$$

where p_0 is pressure on the ground ($z = 0$) and p is pressure at level z. By replacing the logarithm with the first term (higher orders are negligible) of the series approximation,

$$\frac{\delta p_0}{p_0} = \frac{\delta p}{p} + \int_0^z \frac{w}{R(T_0 - \gamma z)} \frac{\partial w}{\partial z}\, dz \,, \tag{12}$$

where δp represents the pressure change due to vertical accelerations.

The vertical acceleration can be expressed as the function

$$w \frac{\partial w}{\partial z} = Az\left(1 - \frac{z}{h}\right) \,, \tag{13}$$

where A is a constant and h is the height at which the vertical velocity reaches a maximum--differentiation of the velocity curves in Fig. 1b have the same shape as the curve described by (13). It is seen from the equation that the acceleration is zero at both the ground and at the level of maximum vertical velocity, as should be expected. After substituting (13) and (12), Levine obtained a positive value for δp_0. However, there were two errors in Levine's paper: 1) the integration of (12) was only up to the level of maximum vertical velocity instead of the top of the cloud and 2) Eq. (13) is representative of the acceleration from the base of the cloud--not from the ground--to the top of the cloud.

Buell (1943a and b) extended Levine's theory to include the integration up to the top of the cloud, but he still defined $z = 0$ as the ground instead of cloud base. By integrating (13) from the "ground" to level z, he found that

$$A = \frac{w^2}{z^2(1 - 2z/3h)} \,.$$

This equation indicates that the top of the cloud (where w is zero) is at a height of $3h/2$. If one remembers that the vertical velocity is a maximum (w_x) at level h, the constant A becomes

$$A = 3w_x^2/h^2$$

By using this definition for A and the approximation

$$\log\left(1 - \frac{\gamma z}{T_0}\right) = -\left[\frac{\gamma z}{T_0} + \frac{1}{2}\left(\frac{\gamma z}{T_0}\right)^2 + \frac{1}{3}\left(\frac{\gamma z}{T_0}\right)^3\right] \,,$$

(12) can be integrated to yield

$$\frac{\delta p_0}{p_0} = \frac{\delta p}{p} + \frac{3w^2z^2}{Rh^2T_0}\left(\frac{1}{2} - \frac{1}{3}\frac{z}{h} + \frac{1}{3}\frac{\gamma z}{T_0} - \frac{1}{4}\frac{\gamma z^2}{hT_0} + \frac{1}{4}\frac{\gamma^2 z^2}{T_0^2} - \frac{1}{5}\frac{\gamma^2 z^2}{hT_0^2}\right). \tag{14}$$

Buell evaluated (14) at $z = 3h/2$, the top of the cloud where δp is zero, using the first three terms in the parenthesis and obtained

$$\frac{\delta p_o}{p_o} = + \frac{27}{8} \frac{h \gamma w_x^2}{R T_o^2} \quad . \tag{15}$$

Again, this is a theoretical confirmation that the dynamic pressure should be positive beneath an updraft.

However, in 1945, Mal and Rao (1945) showed that the terms within the parentheses in (14) should be evaluated in pairs--this can be confirmed by checking the units of the individual terms. Mal and Rao included the fourth term and obtained the confusing result:

$$\frac{\delta p_o}{p_o} = - \frac{27}{64} \frac{h \gamma w^2}{R T_o^2} \quad . \tag{16}$$

Even the negligible fifth and sixth terms in the parenthesis add up to a negative value.

Mal and Rao gave additional evidence for hydrodynamic pressure on the ground being negative. They claimed that the integration constant, k, had been ignored when (13) was integrated. Since k represents air that passes upward through cloud base, they reasoned that, due to the additional momentum, the air should rise an additional amount, Δz, before the vertical motion becomes zero.

If one refers to Fig. 6, the development of the controversy up to this point can be outlined. Levine considered that part of the acceleration curve from $z = 0$ to h and consequently got positive pressure. Buell went all the way up to the top of the cloud, as did Mal and Rao; the proper interpretation of their mathematical results indicates negative pressure on the ground. The heavy curve in the figure is a plot of Eq. (13); an integration of the equation from $z = 0$ to z_T indicates that the positive and negative areas exactly cancel each other. Therefore if one integrates the equation up to $z_T + \Delta z$ to include the effect of the integration constant k, the shaded area in the upper left part of the figure indicates that the integration results in net negative pressure at the ground--contrary to the theoretical findings of this author.

Since the development presented in Section 2 of this paper differs from that of Levine, Buell, and Mal and Rao, it becomes necessary to find an explanation for the difference. First of all let us consider Mal and Rao's interpretation of the integration constant, k. If one uses their explanation that it represents mass flowing upward through the base of the cloud, would it not be more realistic to say that $z = 0$

represents the base of the cloud rather than the ground; in this situation, there should be some acceleration from the ground to the base of the cloud ($z = 0$), as indicated by the shaded area in the bottom of Fig. 6. The inclusion of this shaded area--instead of the one at the top-- in the integration from the ground to top of cloud (z_T) results in positive hydrodynamic pressure on the ground beneath an updraft.

Mal and Rao pointed out that the integration of (13) from $z = 0$ to z_T is zero. If this fact is carried one step further, a predicament becomes apparent. The integrand in (11) or (12) is the same as $\rho w \ \partial w/\partial z$. It has just been stated that the integral of $w \ \partial w/\partial z$ from $z = 0$ to z_T is zero and it is known that the density, ρ , decreases with height; therefore, using density as a weighting factor, it becomes obvious that the integration of $\rho w \ \partial w/\partial z$ from $z = 0$ to z_T results in positive hydrodynamic pressure at $z = 0$. However, using the same basic equation and following the seemingly logical mathematical development outlined above, Mal and Rao obtained an unquestionable negative pressure when integrating from $z = 0$ to z_T .

Implicit error in the Levine-Buell-Mal and Rao model. If the presentation in Section 2 is correct, as is indicated by the density weighting-factor argument, then the mathematical presentation of Levine, Buell, and Mal and Rao must contain a fallacy. After carefully checking and rechecking the mathematic manipulations, the only error was found to be at the very beginning: Eq. (10) is not the proper result when (8) and (9) are combined. At first glance it would appear that one is simply subtracting hydrostatic pressure from the total pressure to get the hydrodynamic component. However, let us consider it from a different point of view. The equation for the pressure distribution within the cloud is

$$\frac{1}{p}\frac{\partial p}{\partial z} = -\frac{1}{RT}\left(g + w\frac{\partial w}{\partial z}\right) , \qquad (8')$$

following Levine's initial derivation. The equation for an assumed hydrostatic environment is

$$\frac{1}{p_s'}\frac{\partial p_s'}{\partial z} = -\frac{g}{R'T'} , \qquad (9')$$

where the prime indicates an environmental value. After subtracting (9') from (8'), we have

$$\frac{1}{p}\frac{\partial p}{\partial z} - \frac{1}{p_s'}\frac{\partial p_s'}{\partial z} = -\frac{g(R'T' - RT)}{RR'TT'} - \frac{w}{RT}\frac{\partial w}{\partial z} . \qquad (10')$$

By comparing this with (10), it is noted that they differ by a buoyancy term (first term on the right side of (10')).

Following through with the integration of (10') from the ground ($z = 0$) to level z ,

$$\log\frac{p_0}{p} - \log\frac{p_0'}{p'} = \log\frac{p_0}{p_0'} - \log\frac{p}{p'} = \int_0^z \left(-\frac{g\,\Delta(RT)}{RR'TT'} - \frac{w}{RT}\frac{\partial w}{\partial z}\right) dz \,, \qquad (11')$$

where Δ is defined as cloud value less environmental value at same level and where the subscript for hydrostatic pressure has been dropped; the environmental total pressure is assumed to be entirely hydrostatic. Using the first term of the series approximation for a logarithm,

$$\log\frac{p_0}{p_0'} = \frac{p_0 - p_0'}{p_0'} \equiv \frac{\Delta p_0}{p_0'}$$

and

$$\log\frac{p}{p'} = \frac{p - p'}{p'} \equiv \frac{\Delta p}{p'}$$

which expresses the ratio of the differential total pressure (as defined in Section 2) to the undisturbed total pressure (or total pressure in the environment at the same level). Equation (11') now becomes

$$\frac{\Delta p_0}{p_0'} = \frac{\Delta p}{p'} + \int_0^z \frac{w}{RT}\frac{\partial w}{\partial z}\,dz - \int_0^z \frac{g\,\Delta(RT)}{RR'TT'}\,dz \,. \qquad (12')$$

A comparison of (12) and (12'), where δ and Δ have identical meanings, reveals that δp_0 represents the change in <u>total</u> pressure on the ground due to vertical accelerations. It is seen that (12) is an incomplete equation, which, in reality, attempts to compute the total pressure change solely from vertical accelerations. In order to compute hydrodynamic pressure, hydrostatic pressure would have to be subtracted from both sides of the equation; this would result in the inequality of hydrodynamic pressure being equal to hydrodynamic minus hydrostatic pressure. Therefore, the final equations obtained by Levine, Buell, and Mal and Rao are meaningless. It should be remembered that they also were operating under the erroneous assumption that the pressure nose is caused by upward accelerations.

<u>Schaffer's approach</u>. In 1947, the South African Schaffer (1947) joined the controversy. He placed little reliance on the results obtained by the above authors,

feeling that too much depended upon an arbitrarily assumed equation (Eq. (13)) and that the results were dependent on the small differences between large terms. An equation similar to (7) was presented for determining "a kind of 'dynamic pressure'" associated with the vertical component of velocity.

Schaffer represented the three-dimensional flow in and below a cloud with stream tubes. Beneath the center of a cloud having a maximum vertical velocity of 10 m sec^{-1}, he computed a dynamic pressure of + 0.2 mb. Using the stream-tube model, he concluded that a region of low pressure surrounded the local high--the low pressure being a consequence of diverging air aloft having a greater contribution to the dynamic pressure than the converging air at lower levels. Schaffer acknowledged that the pressure nose could be due to the additional influence of "temperature changes associated with falling rain (besides several other causes)".

Contribution of Mull and Rao. In a review of explanations for pressure changes in thunderstorms, Mull and Rao (1950) discussed the findings of the above authors. They correctly pointed out the implicit fallacies in the work of Levine, Buell and Mal and Rao. After indicating that Schaffer had obtained positive dynamic pressures on the ground, they summarily discounted it by stating: "This method of computing the dynamic effect cannot explain for the observed pressure rise in thunderstorms." It appears that their main complaint was that he did not consider the density difference between cloud and environment.

Mull and Rao then proceeded to correctly develop an equation which is analogous to (5)-(7), noting that recorded pressure is the resultant of "static pressure" and the "dynamic effect". They did not attempt to evaluate the equation, but did state that it is to be shown that Δp is positive, and not just that Δp_d is positive, beneath an updraft.

Decline of the controversy. Bleeker and Andre (1950), working with Thunderstorm Project data (U.S. Weather Bureau, 1949), were able to explain the pressure nose as being directly related to the total rainfall. Therefore when they referred to the above papers (except Mull and Rao, 1950), they stated: "It is not believed that this sudden change (with appearance of pressure nose) can be explained by applying the types of calculations offered in the papers cited." As it turns out they were quite right, since the dynamic contribution to the pressure nose is of secondary importance (see Fig. 4). As also mentioned by Mull and Rao, Bleeker and Andre were critical of the neglecting of horizontal variations of density.

Based on the comments of Mull and Rao (1950) and Bleeker and Andre (1950), Schaffer (1952) had some additional remarks to make about his 1947 paper. He started with the complete equation (similar to (8)) in order to show the influence of density on the surface pressure. However he apparently did not realize that the criticisms of Mull and Rao and of Bleeker and Andre were directed toward the need for computing both the hydrostatic and hydrodynamic parts of the total pressure. In the mathematical manipulations of the original equation, the hydrostatic part of it was dropped; since the remaining hydrodynamic term had density in it, apparently he felt that he had dealt with the criticisms of the above authors. Again, from a qualitative discussion, he considered there to be negligible pressure change beneath the center of an updraft and a drop in pressure beneath the edges, such that there is a relative rise near the center.

Schaffer also considered the effect of a precipitation downdraft on the surface pressure--the only one of the controversy participants to do so. A column of air cooled rapidly by cold and evaporating rain will contract. Air converging into the column aloft results in a region of low pressure on the ground, surrounding the downdraft. In the descending air, $w(\partial w/\partial z)\,dz$ decreases with time and the density increases such that the integral of $\rho w(\partial w/\partial z)\,dz$ produces a small change on the ground. However as the air decelerates near the ground, both increasing density and increasing $\partial w/\partial z$ lead to a steep rise in the surface pressure--here again there appears to be the misconception that density is synonymous with hydrostatic pressure. (The computations presented for the downdraft in Fig. 3 show a very slow decrease of hydrodynamic pressure with descent, but near the ground the rapid increase is sufficient to make the hydrodynamic pressure positive.)

It has been seen that, except for part of Schaffer's (1952) discussion, the authors discussed above attempted to show that the pressure nose (associated with thunderstorms) is due primarily to upward accelerations of air. Therefore the controversy did not consider the problem of whether the pressure nose is due to hydrostatic or hydrodynamic pressure or whether the nose is caused by updrafts or by downdrafts, but simply whether the hydrodynamic pressure beneath updrafts is positive or negative. All authors give the impression that vertical accelerations have a greater influence on the surface pressure than do variations in density above the recording station.

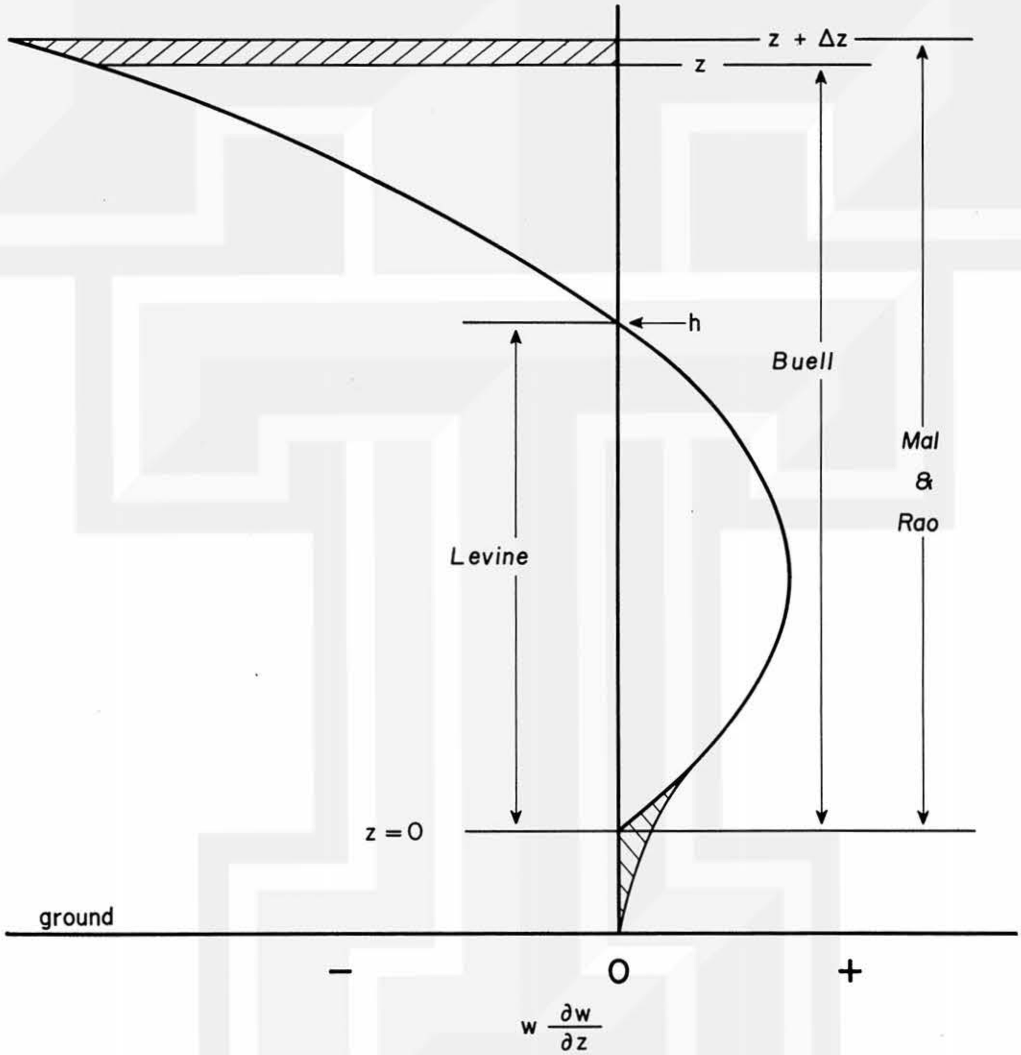

Fig. 6. Limits of integration used by various authors in attempting to obtain the pressure on the ground from vertical accelerations within an updraft.

4. Summary and Conclusions

Due to a general misunderstanding of what the so-called "Thunderstorm-High Controversy" was all about, it was decided to investigate the situation. As is not obvious from a casual reading of the papers involved (especially the earlier ones), the authors were attempting to show that the "nose" of high pressure that forms beneath thunderstorms is due to upward vertical accelerations within the clouds. The papers of Levine (1942) and Buell (1943a and b) indicate high pressure on the ground beneath upward accelerating air; Mal and Rao (1945) show that, due to an error in considering too few terms in a series expansion, dynamic pressure should be negative beneath an updraft. In reality the dynamic pressure should be positive. An analysis of the above papers reveals that a basic equation is not derived properly, so the results obtained are irrelevant.

From a qualitative point of view, Schaffer (1947) was able to show that there should be high dynamic pressure beneath the center of the updraft with lower pressure surrounding the high.

Mull and Rao (1950) pointed out the fallacy in the earlier papers (1942-1945) and presented the proper equation for computing the (total) pressure that appears on a barograph. Schaffer (1952) approached the problem again, this time starting with the equation for total pressure. However in the mathematical manipulations the hydrostatic part of the equation was dropped and he again obtained a high pressure area (apparently thinking that it was high total pressure) beneath the center of the updraft. Being the first of the authors to do so, Schaffer also considered the effect of a precipitation downdraft on the surface pressure (really hydrodynamic pressure only).

In Section 2, the author has made his contribution to the controversy by showing 1) that the hydrodynamic pressure beneath an updraft is positive but that the hydrostatic and total pressures are negative, 2) that the pressure nose is caused by a precipitation downdraft which results in high positive hydrostatic pressure and positive hydrodynamic pressure of smaller magnitude, and 3) that the pressure dome is due to both the outflow of evaporationally cooled air that descended with the precipitation and the ascending air surrounding the downdraft within the cloud. Except within the pressure dome itself, the magnitude of hydrodynamic pressure is only one-third to one-half of the magnitude of the differential hydrostatic pressure.

The author also has discussed the role of pressure gradients on horizontal and vertical motions. It has been shown that for convective circulations, horizontal motions are governed exclusively by the horizontal gradient of the total pressure and vertical motions by the vertical gradient of the hydrodynamic pressure. Without using these considerations, it would be impossible to explain the dynamics that come into play in cumulus and cumulonimbus clouds.

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