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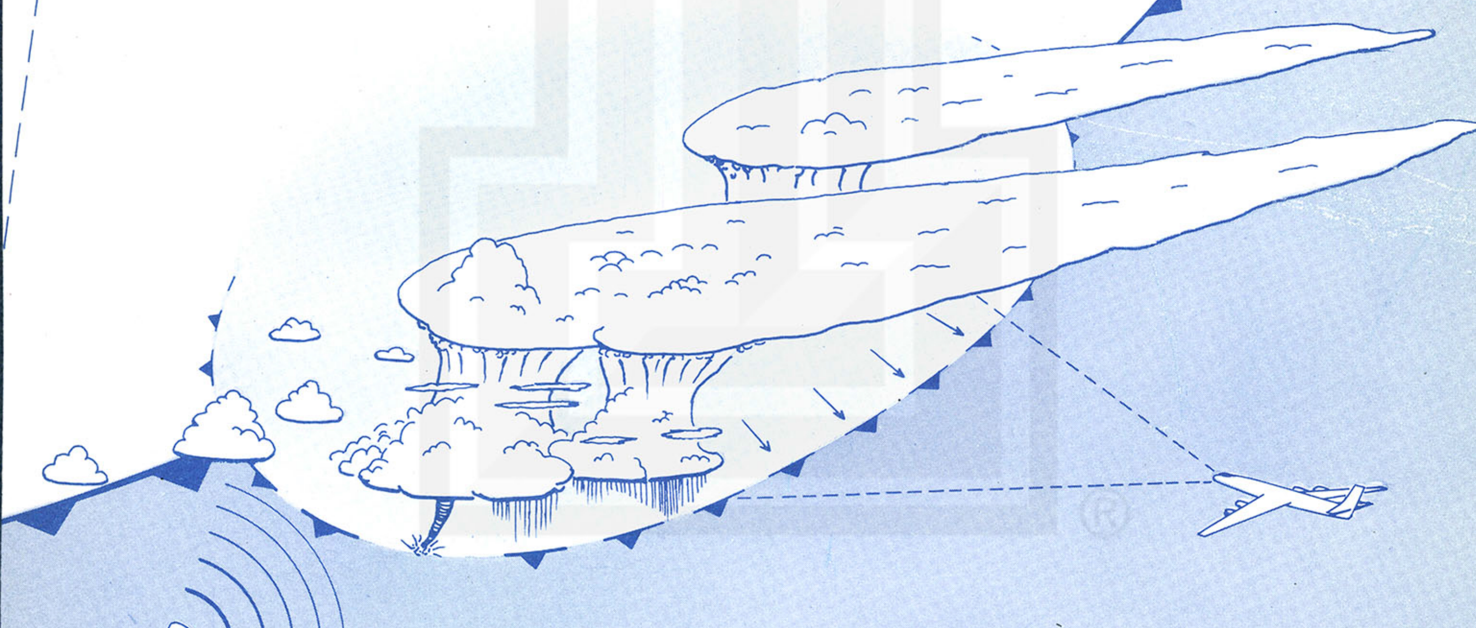
SATELLITE & MESOMETEOROLOGY RESEARCH PROJECT

Department of the Geophysical Sciences
The University of Chicago

ON THE DETERMINATION OF EXCHANGE COEFFICIENTS: PART II - ROTATING AND NONROTATING CONVECTIVE CURRENTS

by

Rodger A. Brown



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SMRP No. 54

On the Determination of Exchange Coefficients:

Part II - Rotating and Nonrotating Convective Currents

ERRATA

Page 26. Richardson's data point for molecular diffusion was inadvertently dropped in the drafting of Fig. 11; the data point is $K = 1.7 \times 10^{-5}\ m^2\ sec^{-1}$, $D = 5.0 \times 10^{-4}\ m$.

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ON THE DETERMINATION OF EXCHANGE COEFFICIENTS: PART II - ROTATING AND NONROTATING CONVECTIVE CURRENTS

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ABSTRACT

A set of equations is proposed for computing eddy viscosity and diffusion coefficients within rotating and nonrotating convective currents. The equations are especially well suited for numerical simulations because the only parameters upon which they depend are the three components of wind velocity. Since the equations are based more on intuition than physical arguments, they are evaluated for simplified models in an attempt to show that they have an implicit physical basis. The physical validity of the equations becomes most obvious when the computed values are compared with those for an actual cumulonimbus and for noncloud data. One outcome of this study is the revision of a "universal" relation originally proposed by Richardson (1926), whereby the exchange coefficient (K) for various scales of atmospheric motion can be determined from a characteristic length (l), which is a measure of the degree of turbulence; the revised relation is

$$K = 1.3 \times 10^{-2} cm^{2/5} sec^{-1} \, l^{8/5} \quad (cgs)$$

or

$$K = 2 \times 10^{-3} m^{2/5} sec^{-1} \, l^{8/5} \quad (mks) \quad .$$

1. Introduction

In Part I of this paper (Brown, 1965), it was shown that one of the proposed sets of equations for determining exchange coefficients gives reasonable results for evaluating the eddy viscosity coefficient for vertical motion in cumuli and cumulonimbi. In Part II, the complete set of equations will be applied to both rotating and nonrotating convective currents. Some of the material presented in Part I will be repeated here in order to give internal consistency to this part.

In the past few years, investigators in the field of cloud dynamics have been

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able to make use of electronic computers to solve the otherwise unmanageable nonlinear differential equations. Malkus and Witt (1959) were among the first to take advantage of computers to study the evolution of a dry convective bubble by using a two-dimensional (x, z) network of grid points. Based on their successful simulation of the early stages of growth, others (see, e.g., Chou, 1962; Lilly, 1962, 1964; Ogura, 1963) have used similar techniques in order to reproduce the general growth features of small cumulus clouds. As computers become faster and have much larger storage facilities, it is inevitable that the growing stage, and eventually the entire life cycle, of cumulonimbi will be simulated.

In order to take turbulent mixing into account in their numerical experiments, investigators have included exchange coefficients (for eddy viscosity and diffusion) in their equations. For simplicity the coefficients have been assumed to be constant throughout the simulation. This is acceptable for the smaller cumulus clouds; however, when trying to simulate the growth from a small cumulus into the large and powerful cumulonimbus, there is an increase of several orders of magnitude in the exchange coefficients. It becomes quite obvious that to use coefficients that are appropriate for a cumulonimbus would prevent the growth of a cumulus and to use coefficients that are appropriate for a cumulus would allow a cumulonimbus to grow beyond realistic bounds. Therefore it would be desirable to have some means for estimating the proper order of magnitude of the coefficients.

In the following sections equations are presented which can be used to calculate the exchange coefficients in a numerical simulation directly from the parameters that are being computed. Then, using simplified models and data from a study of a cumulonimbus, realistic values for the coefficients are calculated for various scales of convective currents that are found in the atmosphere.

2. Discussion of Exchange Coefficients

The calculation of values for the various eddy exchange coefficients is essentially restricted to the surface boundary layer of the atmosphere. When it comes to the determination of the order of magnitude of the exchange coefficients in convective currents, such as those associated with cumulus and cumulonimbus clouds, it becomes quite obvious that the boundary layer approach is not directly applicable.

Of the exchange coefficients, it appears that the one that has been of most interest to theoretical cloud dynamicists is the eddy viscosity coefficient. In almost all cases, the magnitude of the coefficient has been estimated through intuition and the estimates have been only for cumuli; however, there have been a few cases in which estimates of the coefficient were made using aircraft measurements in a cumulonimbus. The ranges in estimates and measurements as given by various investigators are presented in Table 1. The concensus of opinion is that the viscosity coefficient in cumuli should be of the order of 10 to $10^2 m^2 sec^{-1}$; the coefficient in cumulonimbi should be closer to $10^3 m^2 sec^{-1}$.

In Ogura's (1963) simulation of cumulus growth, the magnitude of the viscosity coefficient was varied, with everything else remaining constant. He found basically no difference in the growth for values of 0 and 4 $m^2 sec^{-1}$. However, the use of 40 $m^2 sec^{-1}$ did produce a difference. Therefore, one can conclude from this that the lower limit of the viscosity coefficient in small cumulus clouds is of the order of 10 $m^2 sec^{-1}$.

In light of the above discussion, it would be advantageous to have a set of equations from which the various exchange coefficients within convective currents can be computed using available data. In general one might expect turbulence to be some sort of function of both the wind speed and the shear normal to the direction of the wind. The dimensions of a kinematic exchange coefficient are $[L^2 T^{-1}]$, where L stands for length and T for time. Dimensional analysis reveals that an exchange coefficient has the same units as the ratio of kinetic energy per unit mass to the normal shear of the wind. Thus, in its general form, the proposed exchange coefficient equation is

$$K = \frac{\frac{1}{2} V^2}{\left| \frac{\partial V}{\partial n} \right|} \qquad (1)$$

where K is an eddy exchange coefficient, V is wind speed in a particular direction and n is in a direction perpendicular to that of the wind. The vertical bars in the

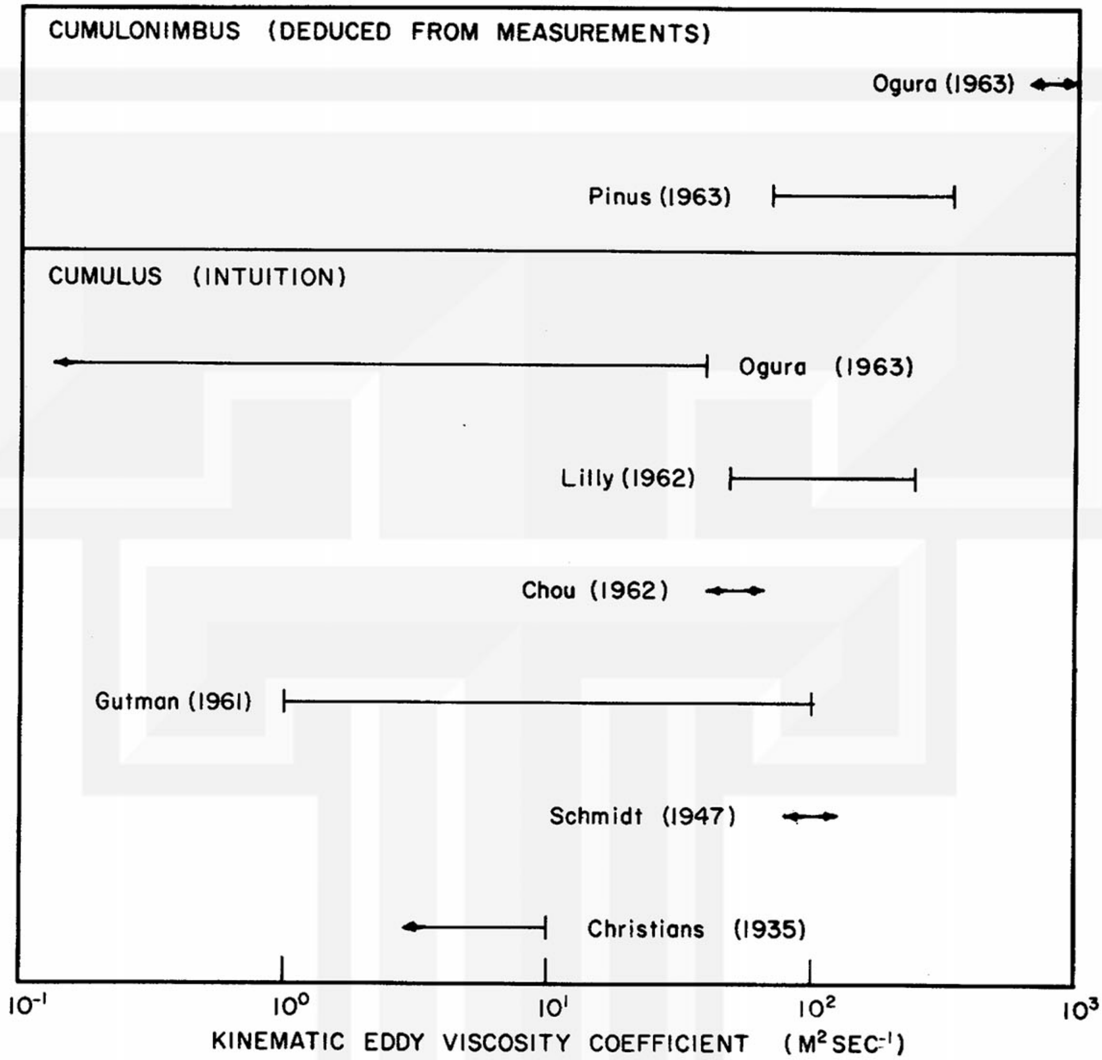

Table 1. Eddy viscosity coefficient in cumulus (intuition or as used in numerical simulation) and cumulonimbus (deduced from aircraft measurements) as mentioned in literature. Vertical line indicates definite limit given by author; arrow indicates order of magnitude.

denominator indicate that the absolute value of the shear should be used in order to make the coefficient positive.

Convective currents found in nature have components of motion in three mutually-perpendicular directions; for instance, vertical, radial, and tangential. This would imply the need for three different viscosity coefficients, an idea which at first might seem strange, but the single viscosity coefficients used in turbulence studies is meant to describe the turbulence only in the "one-dimensional" horizontal wind. The general diffusion equation takes cognizance of the fact that diffusion of momentum, heat, etc. is physically a three-dimensional process. Therefore, it is proposed that the eddy viscosity coefficients for vertical(K_z), radial(K_r) and tangential (K_t) motions be defined, respectively, as

$$K_Z = \frac{\frac{1}{2} w^2}{\left|\frac{\partial w}{\partial r}\right|}, \tag{2}$$

$$K_r = \frac{\frac{1}{2} v_r^2}{\left|\frac{\partial v_r}{\partial z}\right|}, \tag{3}$$

$$K_t = \frac{\frac{1}{2} v_t^2}{\left|\frac{\partial v_t}{\partial z}\right|}, \tag{4}$$

where w, v_r, and v_t are the respective vertical, radial and tangential components of the wind and where r and z represent radial and vertical directions, respectively. The equations include the inherent assumption that convective currents are ideally axially symmetric; therefore, they contain no $\partial/r\partial\theta$ terms.

For turbulence studies it is conventional to have separate coefficients for the eddy diffusion of heat and of water vapor. However, if we view eddy diffusion processes as being due to turbulent motions in the atmosphere, it would seem reasonable to have the same coefficient for the diffusion of both heat and water vapor. The corresponding eddy diffusion coefficient (K_d) would be

$$K_d = \frac{\frac{1}{2}(v_r^2 + v_t^2 + w^2)}{\left|\frac{\partial}{\partial z}(v_r + v_t)\right| + \left|\frac{\partial w}{\partial r}\right|}. \tag{5}$$

Technically, all of the exchange coefficients are diffusion coefficients; however, in this paper, the term <u>viscosity</u> will be used to represent diffusion of momentum and the term <u>diffusion</u> will be understood to represent the diffusion of heat and water vapor.

The set of equations (2) - (5) can be used for computing the exchange coefficients in convective currents, provided the distributions of v_r , v_t , and w are known. This set, therefore, would be best suited for use in a numerical simulation where velocity values at a group of grid points can be computed.

It is important to recognize that Eqs. (2) - (5) are not the end result of a rigorously developed theory of atmospheric turbulence; in fact, the dimensional analysis approach would appear to be devoid of any physical reality. While this may seem to be the case, the following a posteriori verificat on of the physical validity of the equations will be used: if it can be shown

1) that results obtained by using velocity values within a cumulonimbus agree with independently determined coefficients using the same velocity data and

2) that results obtained from both simplified cloud models and an actual cumulonimbus fall in line with values obtained by other investigators for both larger and smaller scale phenomena,

then it will be concluded, without reasonable doubt, that the set of equations have an implicit physical basis.

3. Application of Equations to Rapidly Rotating Convective Currents

The order of magnitude of exchange coefficients in rapidly rotating convective currents (from small eddies and dust devils to large tornadoes) will now be investigated. The most apparent feature of these phenomena is the high tangential velocity. A less obvious feature is the rising motion along the edge of the vortex; there is a question as to whether up- or down-motion occurs in the center. Radial flow is also associated with rotating convective currents; air near the ground converges into the base of a vortex at a much faster rate than it does into the sides of the vortex at higher levels. Since the distribution of radial velocity within a convective vortex is not known and since the radial flow through the side of a vortex is estimated to be negligible compared with the other components of motion, only the exchange coefficients involving vertical and tangential motions will be evaluated.

It will be assumed that all of the vertical motion within a rotating convective current is upward--even though there is increasing evidence to indicate otherwise. Furthermore, it will be assumed that the radial distribution of the updraft has a Gaussian profile. Based on such a profile, the following simplifications will be used to evaluate K_z in Eq. (2): 1) the vertical velocity in the numerator of (2) is equal to the horizontal mean, 2) the horizontal mean (across the column) of the vertical velocity is one-half of the maximum value at the center of the cloud at that level, and 3) the representative shear at that level is equal to the maximum velocity divided by the radius of the column. Figure 1 shows a straight-line approximation to the Gaussian curve that satisfies the above requirements. By substituting the simplifications into (2), the equation reduces to

$$K_z' = \frac{1}{8} \overline{w} D, \qquad (6)$$

where K_z' can be considered as the <u>characteristic</u> eddy viscosity coefficient for a rotating column of diameter D having a mean vertical velocity $\overline{w}$. The word--characteristic--is used in this paper to indicate a prevailing or representative value.

It will be noted that (6) is similar to a conventional mixing-length-theory expression for the eddy viscosity coefficient

$$\nu_e \approx w' l,$$

where ν_e is the kinematic eddy viscosity coefficient, w' is the perturbation velocity which is a measure of the degree of turbulence, and l is a characteristic length of the prevailing eddy. The two equations agree quite closely if D is taken as a measure of the characteristic length and if the perturbation velocity is assumed to be one order of magnitude smaller than the average velocity.

The range in values of K_z' is presented in Fig. 2. The solid portion of each line represents that part of the line which would be valid for natural combinations of diameter and vertical velocity. As might be expected intuitively, the larger and more powerful the dust devil or tornado, the larger the coefficient.

A simplified model for tangential velocity is a bit more difficult. The radial change of v_t can be obtained from the Rankine vortex, where v_t/r is constant between the radius of maximum v_t and the center of the vortex and where $v_t\,r$ is constant at radii greater than that for the maximum velocity. However, (4) requires that the change in v_t with height be specified; this quantity is not theoretically obvious. Fortunately, Hoecker (1960) was able to obtain a vertical cross-section of v_t from measurements made along the edge of the Dallas tornado funnel. Based on corrections, made by Goldman (1965), to eliminate the effect of shock velocities near the center of the lower portion of the tornado (Fig. 3), it was found that within the region where the radius is less than the radius of maximum v_t (region of maximum v_t roughly coincides with edge of visible funnel), the average value of $|\partial v_t/\partial z|$ is approximately 0.2 sec^{-1}. If this value is assumed to be typical for both dust devils and tornadoes, its inclusion in (4) results in

$$K_t' = 2.5 \sec \overline{v_t}^2, \qquad (7)$$

where K_t' is the characteristic viscosity coefficient for a given mean tangential velocity and a vertical shear of 0.2 sec^{-1}. The graphical representation of this equation is given by the solid line in Fig. 4. The dashed lines in the figure represent an arbitrary range in values of K_t' based on vertical shear values of 0.06 and 0.6 sec^{-1}.

In order to evaluate the eddy diffusion coefficient from (5) the following assumptions are made: 1) the radial velocity and vertical shear of the radial velocity are negligible compared to the other values, 2) the remaining terms are evaluated in the same way they were to obtain Eqs. (6) and (7), and 3) the mean tangential velocity is equal to twice the mean vertical velocity (which seems reasonable from the data obtained by Hoecker, (1960). With these assumptions, Eq. (5) becomes

$$K_d' = \frac{5\overline{w}^2 D}{8|\overline{w}| + 0.4 \sec^{-1} D}, \qquad (8)$$

where K_d' is the characteristic eddy diffusion coefficient for vortex of diameter D and with mean vertical and tangential velocities of $\overline{w}$ and $\overline{v}_t (\equiv 2\overline{w})$. The results are plotted in Fig. 5, where again the solid portions of the curves represent values of K_d' that would be found for natural combinations of velocity and diameter.

It has been shown that the exchange coefficients inside vortices increase with both the size of the vortex and the magnitude of the velocity. In addition, intuition suggests that velocity and size of vortex should change in the same direction. If one goes so far as to say that the diameter of the vortex in meters is of the same order of magnitude as the mean vertical velocity in meters per second, then it is possible to combine the data presented in Figs. 2, 4, and 5 into a compact and logical physical picture. The results of such a consideration are given in Table 2, where the additional assumption is made that the mean tangential velocity is equal to twice the mean vertical velocity. The factor $\overline{K}$ was introduced as a mean characteristic exchange coefficient which can be determined for each scale of vortex motion. It is seen that $\overline{K}$ shows a progressive increase as one goes from the scale of small eddies to that of tornadoes.

Table 2. Order of magnitude of various parameters associated with atmospheric vortices ranging from small eddies to tornadoes. A mean characteristic exchange coefficient for each scale of vortex is indicated by $\overline{K}$.

Phenomenon	D (m)	$\overline{w}$ (m sec^{-1})	$\overline{v}_t$ (m sec^{-1})	K'_z (m^{2}sec^{-1})	K'_t (m^{2}sec^{-1})	K'_d (m^{2}sec^{-1})	$\overline{K}$ (m^{2}sec^{-1})
eddy	10^{-1}	10^{-1}	2×10^{-1}	10^{-3}	10^{-1}	10^{-2}	10^{-3} to 10^{-1}
small dust devil	1	1	2	10^{-1}	10	1	10^{-1} to 10
dust devil	10	10	2×10	10	10^3	10^2	10 to 10^3
tornado	10^2 to 10^3	10^2	2×10^2	10^3	10^5	10^4	10^3 to 10^5

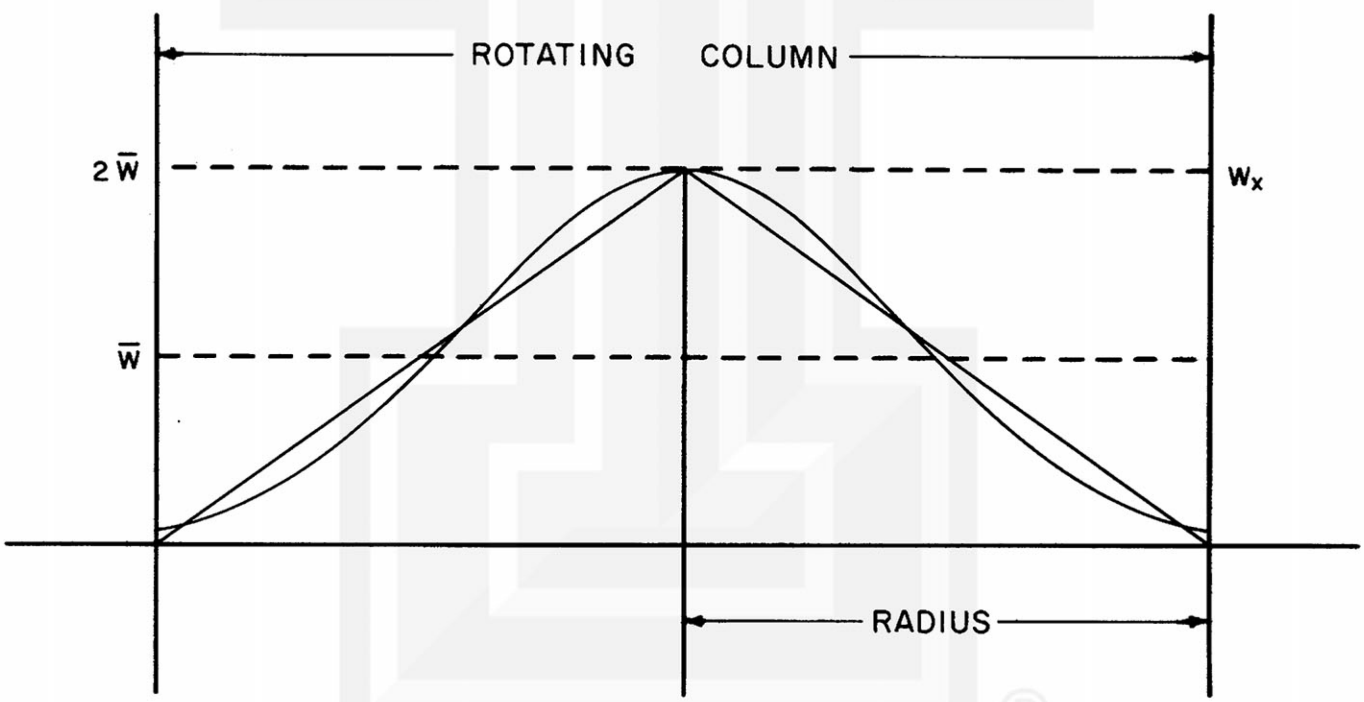

Fig. 1. Example of linear approximation for Gaussian vertical velocity profile. It is constructed such that horizontal mean is one-half of maximum value.

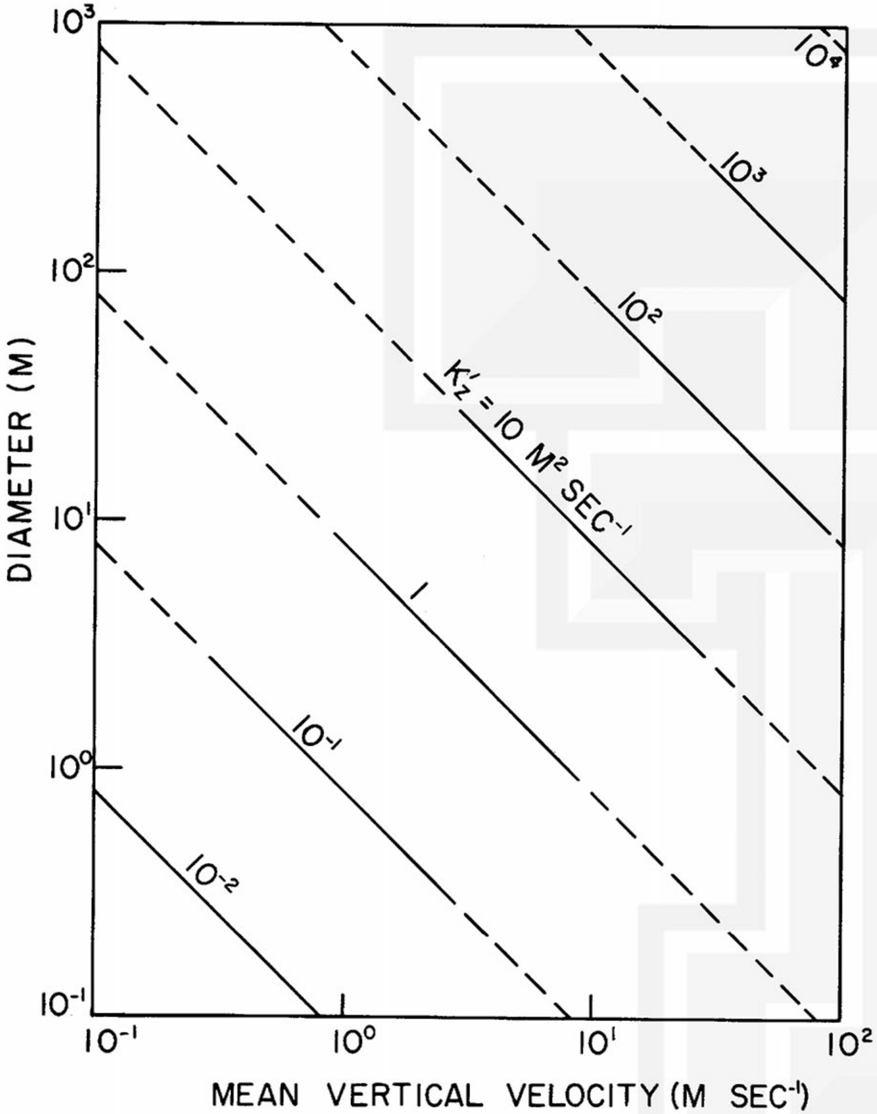

Fig. 2. Characteristic (or representative) eddy viscosity coefficient for vertical motion in a rotating updraft. Solid portion of each line represents that part which would be valid for natural combinations of diameter and vertical velocity.

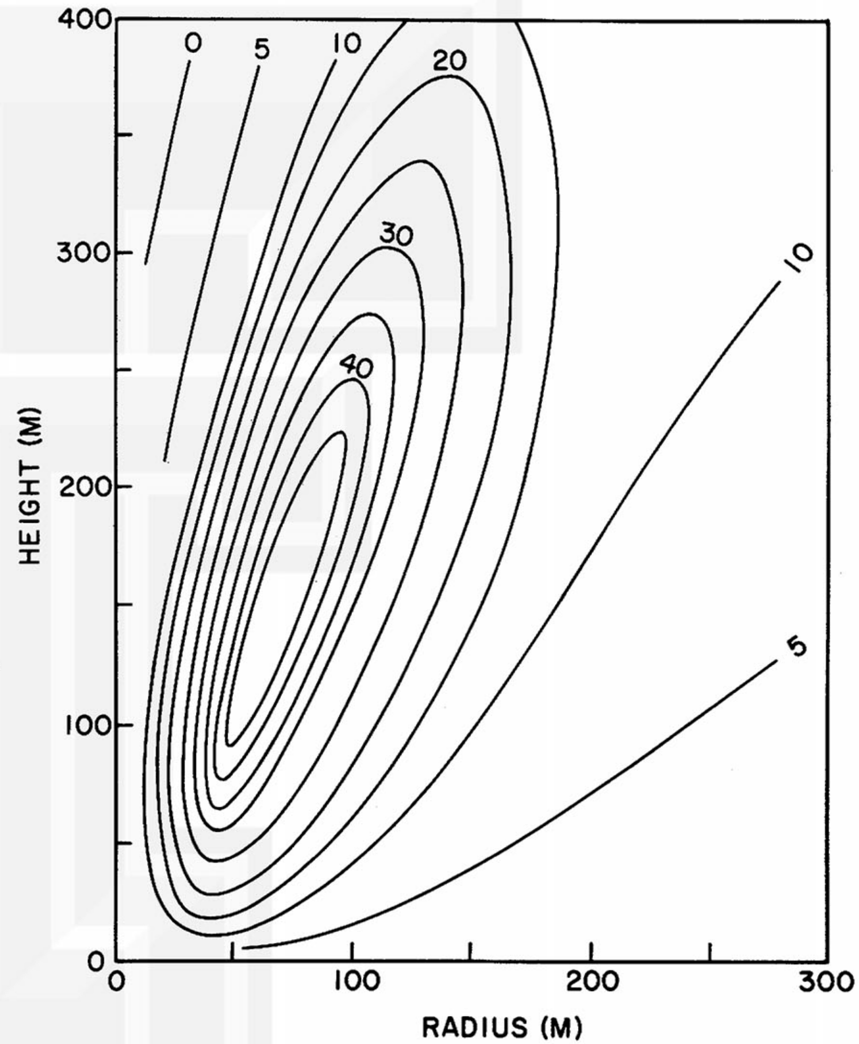

Fig. 3. Isopleths of tangential velocity (m sec^{-1}) found along the edge of the Dallas tornado funnel. Data of Hoecker (1960) corrected by Goldman (1965) to eliminate effect of shock velocities near the center of the lower portion of the tornado.

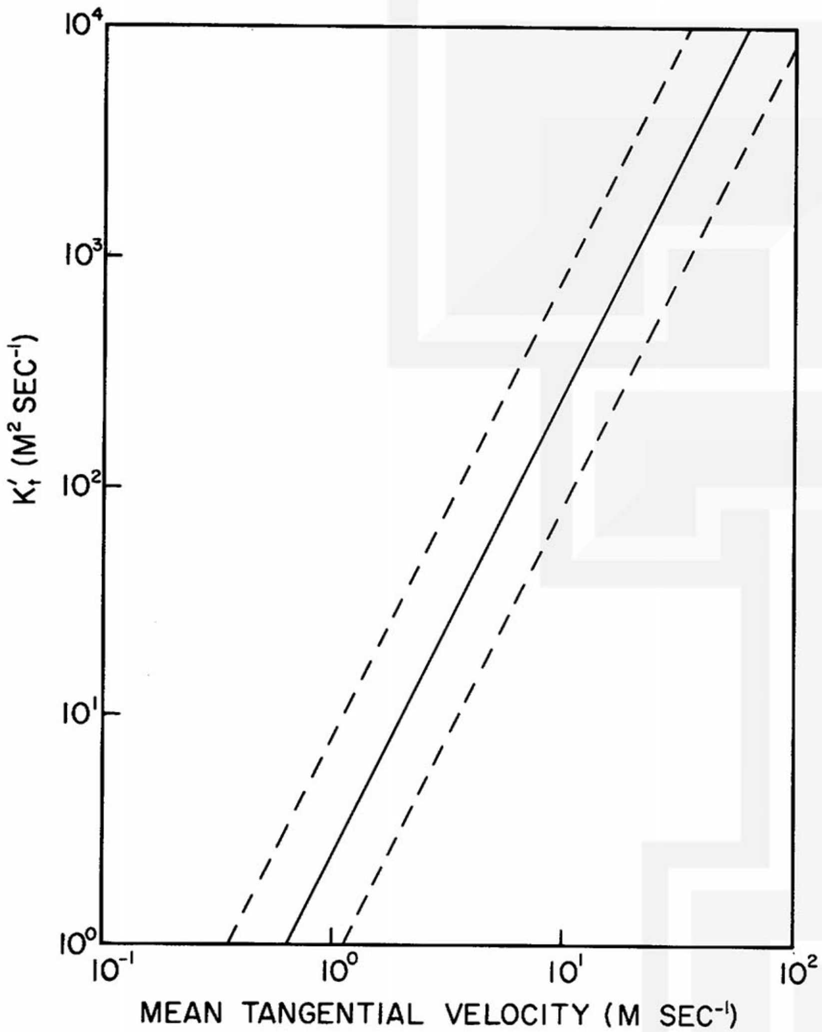

Fig. 4. Characteristic (or representative) eddy viscosity coefficient for tangential motion in a rotating updraft as a function of the mean tangential velocity. Solid line is based on a value of 0.2 sec^{-1} for vertical shear of tangential velocity. The dashed lines are for an arbitrary range in shear values from 0.06 to 0.6 sec^{-1}.

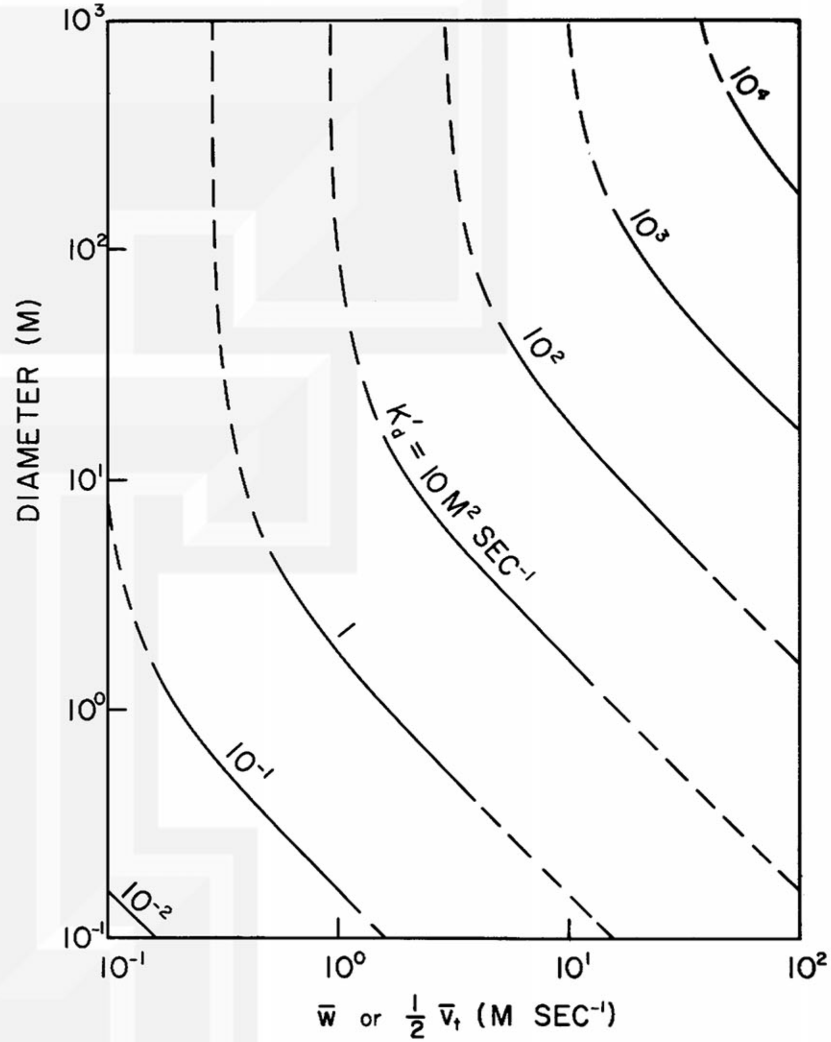

Fig. 5. Characteristic (or representative) eddy diffusion coefficient for a rotating updraft. Solid portion of each line represents that part which would be valid for natural combinations of diameter and vertical or tangential velocity.

4. Application of Equations to Convective Clouds

By applying Eqs. (2) - (5) to convective currents in clouds it will be possible to check the intuitive values given in Table 1. For clouds the most important component of motion is in the vertical, with the radial component being of secondary importance. The tangential component does not occur in small cumuli, but, even though it does start to appear in large cumulus congestus and cumulonimbus clouds, it can be neglected in the larger nontornado-producing clouds in comparison with the other two components of motion. Therefore, in this section we are concerned only with determining the exchange coefficients for vertical and radial motion.

The same assumptions that were used to derive K_z' above will be used here, where we know that the vertical motion in the center <u>is</u> upward during the growing stage. The pertinent equation is

$$K_z' = \frac{1}{8} \overline{w} D \tag{9}$$

The curves of K_z' for a realistic range of $\overline{w}$ and D are plotted in Fig. 6. Except for very large clouds, D can be taken as cloud diameter as well as diameter of the updraft. For small cumulus clouds, an eddy viscosity coefficient of the order of $10^2 m^2 sec^{-1}$ seems to be quite realistic and to be in agreement with the concensus of values shown in Table 1. However, the range of values found by Pinus (1963) for a cumulonimbus several km in diameter and having a mean vertical velocity of about 5 m sec^{-1} are one order of magnitude smaller than those indicated by Fig. 6. Ogura's (1963) estimate from aircraft data is in closer agreement with these theoretical results.

It was mentioned above in conjunction with Table 1 that one could conclude from the numerical simulations of Ogura (1963) that the smallest viscosity coefficient that one might expect in a cumulus cloud is of the order of 10 $m^2 sec^{-1}$. The data presented in Fig. 6 are in total agreement with this conclusion.

The establishment of a simplified model for finding the vertical distribution of radial velocity is necessary before (3) can be evaluated. As a rough approximation, one can assume 1) that there is a linear change in the radial velocity from cloud base to cloud top and 2) that the maximum rate of inflow (cloud base) has the same magnitude as the maximum rate of outflow (cloud top). It is recognized that such a profile is a rather crude approximation; even though it allows for radial conservation of mass, one would expect a weaker convergence in the mid and lower portions of a cloud and a stronger divergence in the upper-most region.

In an attempt to derive an expression for the vertical distribution of radial velocity, Fig. 7 was constructed. Fig. 7a depicts an idealized radial distribution of radial velocity with the coordinates being normalized radial velocity (relative to maximum radial velocity) and normalized radius (relative to radius of cloud). With such a profile, the horizontal mean of the radial velocity turns out to be 70 per cent of the maximum value. Fig. 7b shows, in a normalized presentation, the vertical distribution of radial velocity in various parts of the cloud based on the two assumptions mentioned above. The vertical line indicates that there is no radial motion along the center of the cloud. The sloping solid line represents the change in maximum radial velocity at various heights. As seen by comparison with Fig. 7a, the vertical profile of radial velocity in any part of the cloud should lie in the shaded region. The dashed line represents the vertical distribution for the horizontal mean of the radial velocity; a numerical value for this distribution is needed in the denominator of Eq. (3).

In order to obtain the numerical value, reference must be made to empirical data. Radial velocity data with which the author is most familiar is that found in Brown and Fujita (1965). The data were calculated, using the continuity equation, from vertical velocity data which had been obtained semi-empirically (the procedure is outlined in more detail below). Even though the information is not based on direct measurements, it should be realistic enough for present purposes. The maximum radial velocities found at six different levels are plotted at their respective heights above cloud base (Fig. 8). The dashed line is a mean straight line through the scatter of points. The solid line has the same slope as the dashed one but is symmetric with respect to the level of zero radial velocity; this line corresponds to the solid sloping line in Fig. 7b.

The information is now available for determining a simplified version of Eq. (3). The vertical change of the mean radial velocity according to Figs. 7 and 8 is

$$\left|\frac{\Delta \bar{v}_r}{\Delta z}\right| = \frac{0.7 \times 7.0 \text{ m sec}^{-1}}{7.3 \times 10^3 \text{m}} = 6.7 \times 10^{-4} \text{sec}^{-1}.$$

It is being assumed 1) that all clouds from small cumuli to large cumulonimbi have the same change of radial velocity with height and 2) that the inflow and outflow velocities at cloud base and top, respectively, are roughly proportional to the size of the cloud. Since the only size factor available in this model is height, the relationship between mean radial velocity at cloud base or top ($\overline{V_{r_o}}$) and the vertical extent of the cloud (Z) is

$$\left|\overline{V_{r_o}}\right| = kZ,$$

where, from the data given in Fig. 8, k is

$$k = \frac{0.7 \times 3.5 \text{ m sec}^{-1}}{7.3 \times 10^3 \text{ m}} = 3.35 \times 10^{-4} \text{sec}^{-1}.$$

Now, since a mean radial velocity for the entire cloud is needed, it can be specified as being one-half the magnitude of the radial-mean value at cloud base or

$$\left|\overline{\overline{V_r}}\right| = \frac{1}{2}\left|\overline{V_{r_o}}\right| = 1.68 \times 10^{-4} \text{sec}^{-1} Z,$$

where the double horizontal bar is the average over the entire cloud.

The characteristic eddy viscosity coefficient for radial motion in the simplified model of a cloud is

$$K_r' = \frac{\frac{1}{2}\,\overline{\overline{V_r}}^2}{\left|\frac{\Delta \overline{V_r}}{\Delta z}\right|} = 2.1 \times 10^{-5} \text{ sec}^{-1} Z^2. \tag{10}$$

This equation is plotted in Fig. 9.

If it is assumed that the tangential velocity is negligible in all clouds except the rare tornado-producing ones, the above approximations can be applied to derive a simplified diffusion coefficient; however, instead of assuming that the mean radial velocity is a function of the depth of the cloud, it will be assumed that it is equal to one-tenth of the mean vertical velocity. The characteristic eddy diffusion coefficient is then represented by

$$K_d' = \frac{1.1\,\overline{w}^2 D}{8\,|\overline{w}| + 1.34 \times 10^{-3} \text{sec}^{-1} D}; \tag{11}$$

its range of values are plotted in Fig. 10. Since the relative influence of the radial velocity is so small, Figs. 6 and 10 differ only at the larger diameters.

Table 3 contains a general summary of the magnitude of the exchange of coefficients that could be expected in cumulus and cumulonimbus clouds. The values for the diameter, vertical extent, and vertical and radial components of velocity are quite arbitrary and are used only to determine typical values for the coefficients. For tornado-producing cumulonimbi, which have a fairly high tangential velocity near the axis of rotation, the viscosity coefficient for tangential motion will be an important exchange coefficient; however, for the ordinary clouds, tangential velocities are negligible and the corresponding exchange coefficient need not be determined. In comparison with rotating currents (Table 2), the range in the characteristic coefficients for a particular phenomenon is much smaller for nonrotating updrafts. It should be remembered that the use of the word <u>cloud</u> here assumes that the entire cloud is one updraft.

Table 3. Characteristic magnitudes of exchange coefficients in cumulus (humilis and congestus) and cumulonimbus clouds. Double horizontal bars indicate average throughout entire cloud; D is cloud diameter; Z is vertical extent; $\overline{K}$ is typical range in coefficients.

Cloud type	D (km)	Z (km)	$\overline{\overline{w}}$ (m sec^{-1})	$\overline{\overline{v}}_r$ (m sec^{-1})	K'_z (m^{2}sec^{-1})	K'_r (m^{2}sec^{-1})	K'_d (m^{2}sec^{-1})	$\overline{K}$ (m^{2}sec^{-1})
Cu hu	1	1	1	0.1	10^2	2 x 10	10^2	10 to 10^2
Cu con	3	6	4	0.4	10^3	8 x 10^2	10^3	10^3
Cb	5	10	6	0.6	4 x 10^3	2 x 10^3	4 x 10^3	2-4 x 10^3

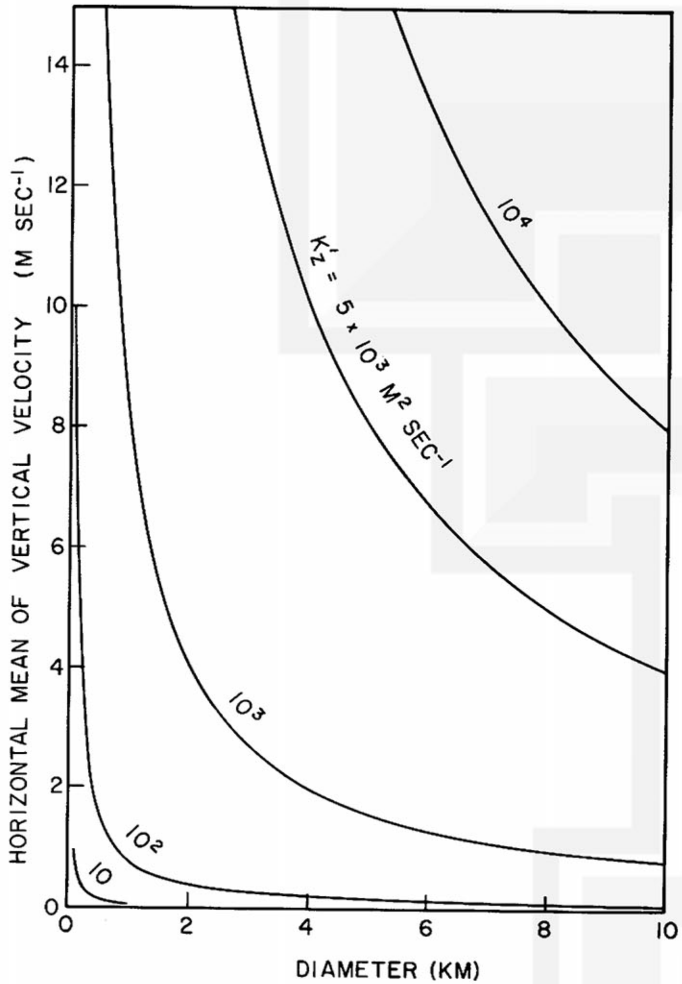

Fig. 6. Characteristic (or representative) eddy viscosity coefficient for vertical motion in updrafts within clouds. Except for very large clouds, the diameter of the updraft (abscissa) can be considered equal to cloud diameter.

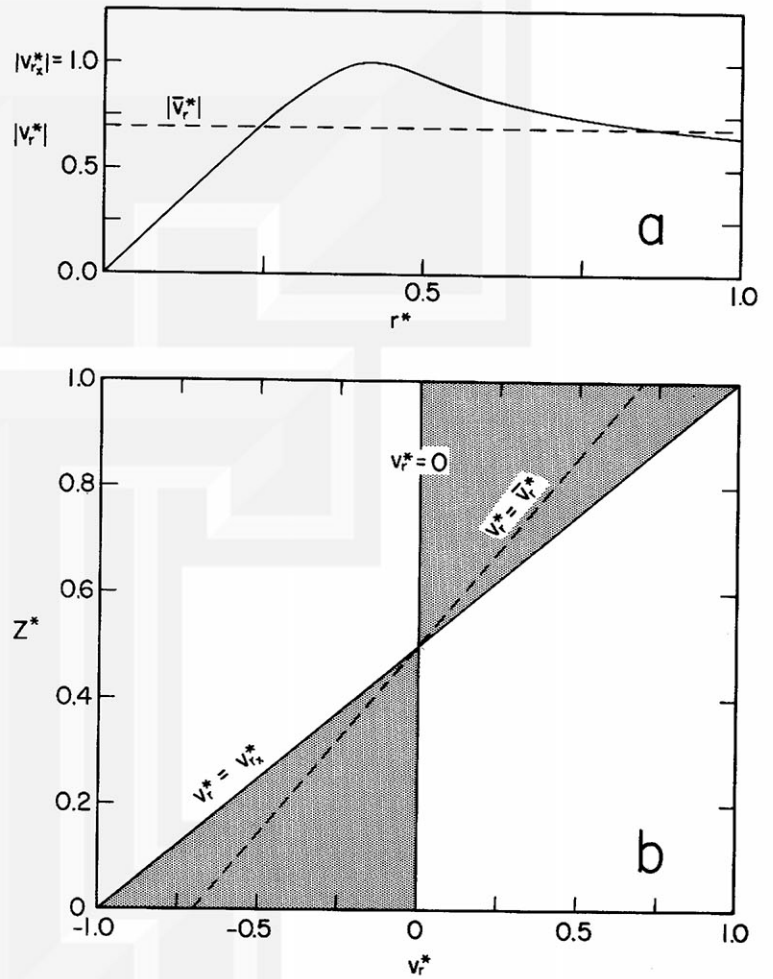

Fig. 7 a. Idealized radial distribution of radial velocity. Superscript star signifies nondimensional number; r is radius; horizontal bar is mean value; subscript x is maximum value; vertical bars represent absolute value.

b. Hypothetical vertical distribution of radial velocity from cloud base to cloud top. Dashed line represents simplified vertical distribution of mean radial velocity.

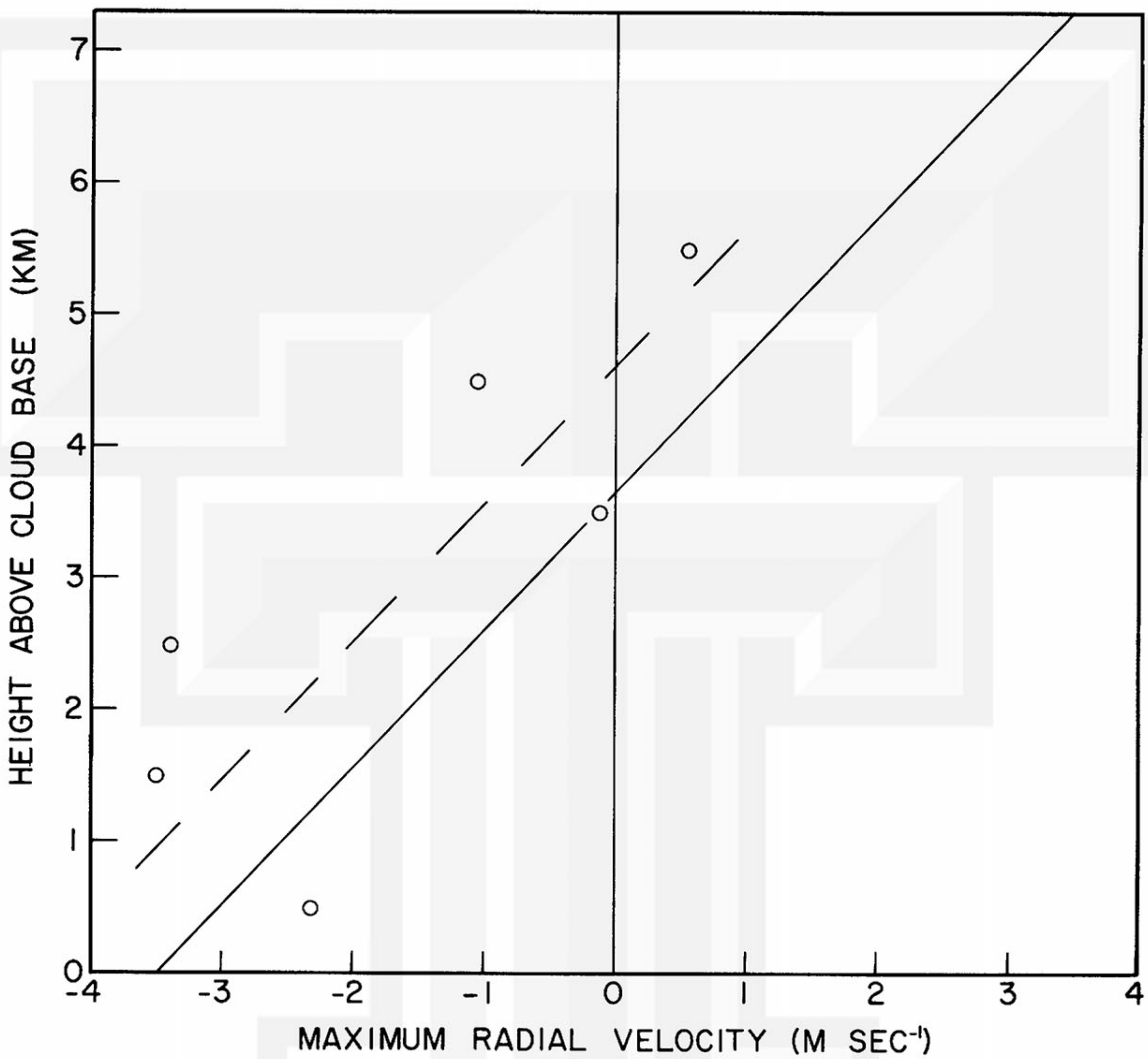

Fig. 8. Maximum radial velocities as a function of height above cloud base. Dashed line is mean of maximum radial velocity values (circles) obtained from Brown and Fujita (1965). Solid line has same slope as dashed one but is symmetric with respect to level of zero radial velocity.

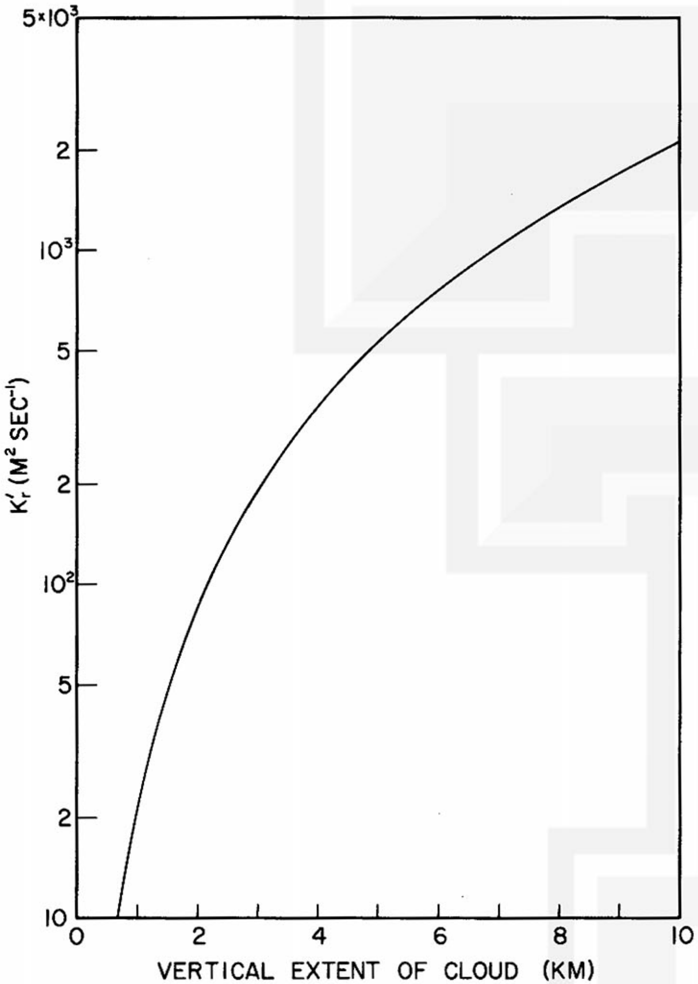

Fig. 9. Characteristic (or representative) eddy viscosity coefficient for radial motion for updrafts (or clouds) with the indicated vertical extents.

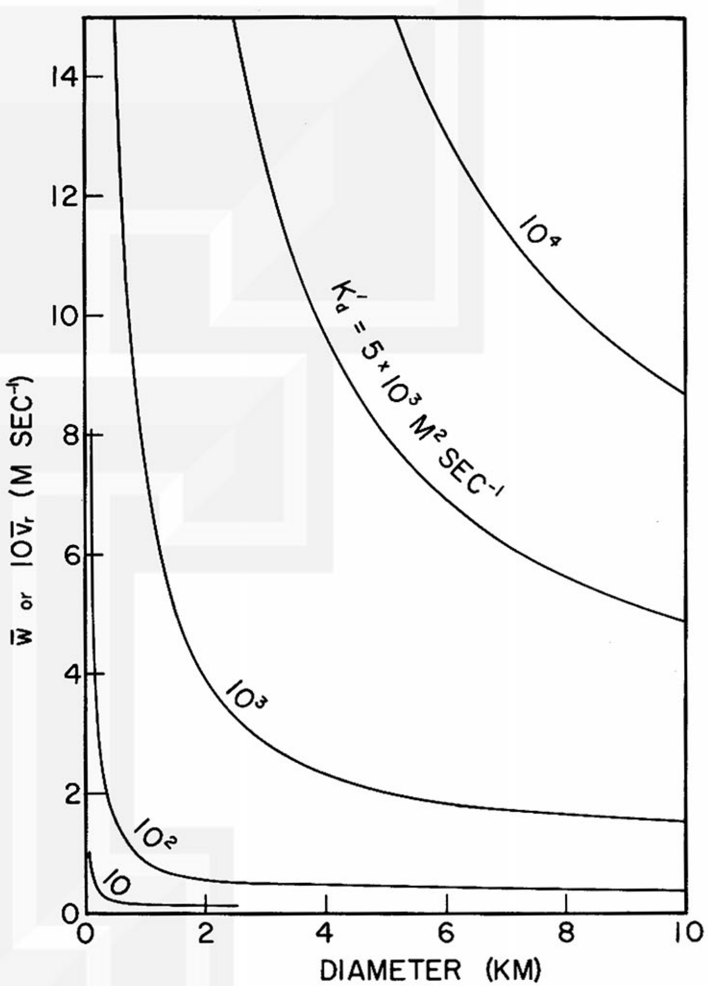

Fig. 10. Characteristic (or representative) eddy diffusion coefficient for updrafts (or clouds, if they consist of single updraft) as a function of draft diameter and vertical or radial velocity.

5. Evaluation of Eddy Exchange Coefficients within a Cumulonimbus

Up to this point, the only arguments presented for the validity of the original set of Eqs. (2) - (5) have been that the viscosity coefficient for vertical motion agrees with the concensus of various estimates and that the coefficients vary in an expected manner. In this section the equations will be tested by comparing them with semi-empirical data from a cumulonimbus, as obtained by Brown and Fujita (1965).

Since the work of Brown and Fujita (1965) has not yet been published in a widely-read periodical, it would be wise to outline their work. In an attempt to combine the flux of mass into the expanding anvil top of a thunderstorm (total anvil mass determined from stereophotogrammetry) with the theoretical vertical velocity (one large updraft assumed) within the cloud prior to the formation of a downdraft (entrainment procedure applied to proximity rawinsonde observation in northern Arizona), they found that there was no direct correspondence between the two. The problem was finally solved by developing a model for a cumulonimbus cloud prior to the formation of a downdraft; even though the lack of a downdraft in a cumulonimbus is contrary to the Byers-Braham (1948) model of a thunderstorm, it is representative of thunderstorms that occur in the arid regions of the southwestern United States. The model cumulonimbus consists of three concentric cylindrical regions which represent the vertical stem of the cloud; the turbulent entrainment of environmental air is greatest in the outer region and decreases to zero in the central region where only the vertical component of motion is found. Using the three-region model, Brown and Fujita proceeded to compute the vertical and radial distributions of vertical velocity within the cumulonimbus as it appeared 10 minutes after the initial formation of the anvil; though the vertical velocities were on the high side--maximum of 30 m sec^{-1}--they did not exceed those found in cumulonimbi. By making use of the continuity equation for axial symmetry, the horizontal distribution of radial velocity was computed from the vertical distribution of vertical velocity at height intervals of one kilometer (the points plotted in Fig. 8 were obtained from the resulting curves).

By recognizing the equivalence of the entrainment concept and turbulent exchange processes, Brown and Fujita replaced the entrainment expression in their equation of vertical motion with the conventional eddy viscosity term

$$\nu_e \nabla^2 w$$

where ν_e is the eddy viscosity coefficient and ∇^2 is the Laplacian operator. In this way, with everything else known or able to be calculated from the computed data, the equation could be solved for ν_e . At 66 data points (horizontal interval 0.4 km and vertical interval 1.0 km) within the stem of the cloud below anvil level, the resulting values of ν_e ranged from 1.7×10^2 to 2.0×10^4 m^2sec^{-1}, with a median value of 3.2×10^3 m^2sec^{-1}. At the time of calculation the cloud was 7.3 km tall, the stem (and assumed updraft) was 4.0 km wide and the mean vertical velocity within the stem (at the same 66 data points) was 9.3 m sec^{-1}; from this, Fig. 6 indicates that the characteristic eddy viscosity coefficient for vertical motion should be 4.7×10^3 m^2 sec^{-1}. Therefore, in the mean, there is strong evidence that at least Eq. (2) is physically valid.

As an even better idea of the validity of (2), the equation should be evaluated at the same 66 data points; however, since the denominator becomes zero at the center of the cloud, (2) can be evaluated only at the other 60 data points. The resulting values ranged from 2.0×10 to 1.7×10^4 m^2sec^{-1}, with a median value of 1.4×10^3 m^2sec^{-1}. These data for the eddy viscosity coefficient for vertical motion are summarized in Table 4. The consistency of both the median and range in the data is extremely encouraging.

Table 4. Range in, and median values of, eddy viscosity coefficient for vertical motion within a cumulonimbus based on the semi-empirical data of Brown and Fujita (1965) and on the theoretical equations proposed herein.

	Eddy Viscosity Coefficient (m^2sec^{-1})		
Equation	Lowest	Median	Highest
ν_e (66 points)	$1.7x10^2$	$3.2x10^3$	$2.0x10^4$
K_z' (D , $\overline{w}$-Fig. 6)	---	$4.7x10^3$	---
ν_e (60 points)	$1.7x10^2$	$3.0x10^3$	$2.0x10^4$
K_z (60 points)	$0.2x10^2$	$1.4x10^3$	$1.7x10^4$

While it will not be possible to compare the coefficient for radial motion with independently derived values, it will be possible to evaluate Eq. (3) using the radial velocity distribution given by Brown and Fujita and then to compare the median value with the characteristic value given in Fig. 9. In order to evaluate (3) the problem still exists concerning $|\partial v_r/\partial z|$; again the denominator is zero along the central axis of the cloud. In order to simplify the problem, the vertical shear of the radial velocity is taken as a constant value with height for each radial distance. The shear value is determined in the same way as it was in Fig. 8; at each of the 60 data points the actual value of radial velocity is used. As a result of evaluating Eq. (3), the range was from 1.6×10^0 to 6.7×10^3 $m^2 sec^{-1}$; the median value was 1.2×10^3 m^2 sec^{-1}. For a cloud 7.3 km tall, Fig. 9 predicts that the characteristic viscosity coefficient for radial motion should be 1.1×10^3 $m^2 sec^{-1}$. Even though basic information from the same cumulonimbus was used in order to derive Eq. (10) and Fig. 9, identical information was not used in the evaluation of (3). Therefore, it can be argued that we again have an indication, though less rigorous, that the set of Eqs. (2) - (5) have physical validity.

By assuming that tangential velocities within a cloud are negligible compared to radial and vertical velocities, it is possible to use the above-mentioned data to compute the diffusion coefficient from (5). The values of the coefficient were calculated for each of the 60 data points used for the viscosity coefficients. The resulting values ranged from 2.9 x 10 to 1.6×10^4 $m^2 sec^{-1}$ and the median was 1.4×10^3 $m^2 sec^{-1}$. A comparison of the various exchange coefficients within the cumulonimbus is made in the next section and the values are tabulated in Table 5.

Since data are now available, a check should be made of the ratio of mean radial velocity to mean vertical velocity in order to test the realistic aspects of the assumption used in the derivation of (11). The mean radial velocity at all 66 data points within the cloud stem was 1.3 m sec^{-1}. As mentioned above the mean vertical velocity was 9.3 m sec^{-1}. So $\overline{\overline{w}}$ is only 7.0 times larger than $\overline{\overline{v_r}}$, instead of being the assumed 10 times larger. However, the assumption that $\overline{\overline{w}}$ was 10 times larger underestimates the characteristic diffusion coefficient by only 4 per cent and therefore the assumption is quite valid. With this small correction factor taken into account, the characteristic eddy diffusion coefficient for the above cloud is 5.0×10^3 $m^2 sec^{-1}$; in the case of both this and the other coefficients, the characteristic coefficients have been larger than the median coefficients but all definitely have been of the same order of magnitude.

6. Derivation of Empirical Relations

All of the coefficients obtained in the preceding sections are plotted in the log-log presentation found in Fig. 11; also plotted are data points used by Richardson (1926) in deriving the well-known empirical relation

$$K = 0.2 \ \mathrm{cm}^{2/3} \ \mathrm{sec}^{-1} \ l^{4/3} \ (\mathrm{cgs}), \tag{12}$$

where l is a characteristic length representing the separation of two entities which initially were side-by-side. Therefore, as time goes on and the two entities become farther apart due to the influence of larger turbulent eddies, the magnitude of the coefficient increases. In order to plot the data points in the figure, it was assumed that l represents the mean diameter of an updraft (or cloud).

Even though Richardson's data sources will not be mentioned here, it would be advisable to indicate what the coefficients represent. The smallest coefficient is for the molecular diffusion of oxygen into nitrogen, the largest is for "diffusion due to cyclones regarded as deviations from the mean circulation of the latitude," and the others are obtained from the mean variation of wind with height (l being the vertical separation of anemometers) and from the motion of balloons and volcanic ash (l being the mean of the height of the object above ground and the vertical displacement between two sightings). Considering the variety in the method of measurement, it is surprising that Richardson could find any consistency between the data points.

The dashed line in Fig. 11 is the one Richardson drew through his points and is described by (12). The solid lines for

$$K = \left(\frac{D}{100}\right)^2 \ (\mathrm{mks}) \tag{13}$$

and

$$K = D^2 \ (\mathrm{mks}) \tag{14}$$

represent simple relations for the change in coefficient with diameter for nonrotating and rotating updrafts, respectively. While the data points for nonrotating updrafts (clouds) remain close to the line, there is a one order of magnitude scatter in the coefficients for vortices. Therefore, Eq. (14) can be used to obtain only a very crude idea of the order of magnitude of exchange coefficients in rapidly rotating convective currents.

A summary of the coefficients computed for the cumulonimbus in the previous section and the coefficients indicated by (12) and (13) are presented in Table 5. It is seen that Richardson's equation underestimates and that (13) slightly overestimates the median values for the three coefficients.

Table 5. Summary of exchange coefficients for the cumulonimbus, with median values computed from Eqs. (12) and (13).

Exchange Coefficient	Lowest	Median ($m^2 sec^{-1}$)	Highest
K_z (60 points)	2.0 x 10	1.4 x 10^3	1.7 x 10^4
K_r (60 points)	0.2 x 10	1.2 x 10^3	0.7 x 10^4
K_d (60 points)	2.9 x 10	1.4 x 10^3	1.6 x 10^4
Richardson's Eq. (l = 4 km)	---	0.6 x 10^3	---
$K = (D/100^2)$ (D = 4km)	---	1.6 x 10^3	---

It is noted in Fig. 11 that Richardson's lowest data point, which is the only one representing molecular diffusion, departs markedly from a straight-line trend of the remaining eddy diffusion points. Though it may be unethical to tamper with another's data, it is felt that, in light of the data computed in this paper and the role of the molecular diffusion point in biasing the slope of Richardson's line, a re-evaluation of Eq. (12) should be made.

Figure 12 shows a plot of Richardson's eddy values, the mean of the log of the exchange coefficients (Z, R, F) from Fig. 11 for each of the three cloud types (see Table 3 for tabulated values), and a mean of the three coefficients computed for the cumulonimbus (see Table 5). The sloping line was computed by the method of least squares. The closeness of data points to the line is extremely encouraging from two points of view:

1) even though Eqs. (2) to (5) had no rigid physical basis, the fact that the computed values--especially the one for the actual cumulonimbus--fall so closely in line with independent values leads one to strongly believe that the equations are capable of predicting extremely realistic values

for the various exchange coefficients; and

2) there seems to be a "universal" relationship between exchange coefficients and a "characteristic length", which can be considered as the predominant scale of turbulence in a given situation--no mater whether one is concerned with general turbulence in the air, turbulence within a cloud, or the deviation of cyclones from the mean circulation of a given latitude.

A re-evaluation of Richardson's data together with the data computed above results in the following equivalent universal relations:

$$K = 1.3 \times 10^{-2} \mathrm{cm}^{2/5} \sec^{-1} l^{8/5} \text{ (cgs)} \qquad (15\text{ a})$$

or

$$K = 2 \times 10^{-3} \mathrm{m}^{2/5} \sec^{-1} l^{8/5} \text{ (mks)}, \qquad (15\text{ b})$$

where K can be considered as representing any of the coefficients because, in nonrotating air, the various coefficients appear to be of the same order of magnitude; the characteristic length (l) concept has been used here without considering the physical processes involved--in the case of clouds, l can be taken as the diameter of nonrotating updraft or downdraft cells.

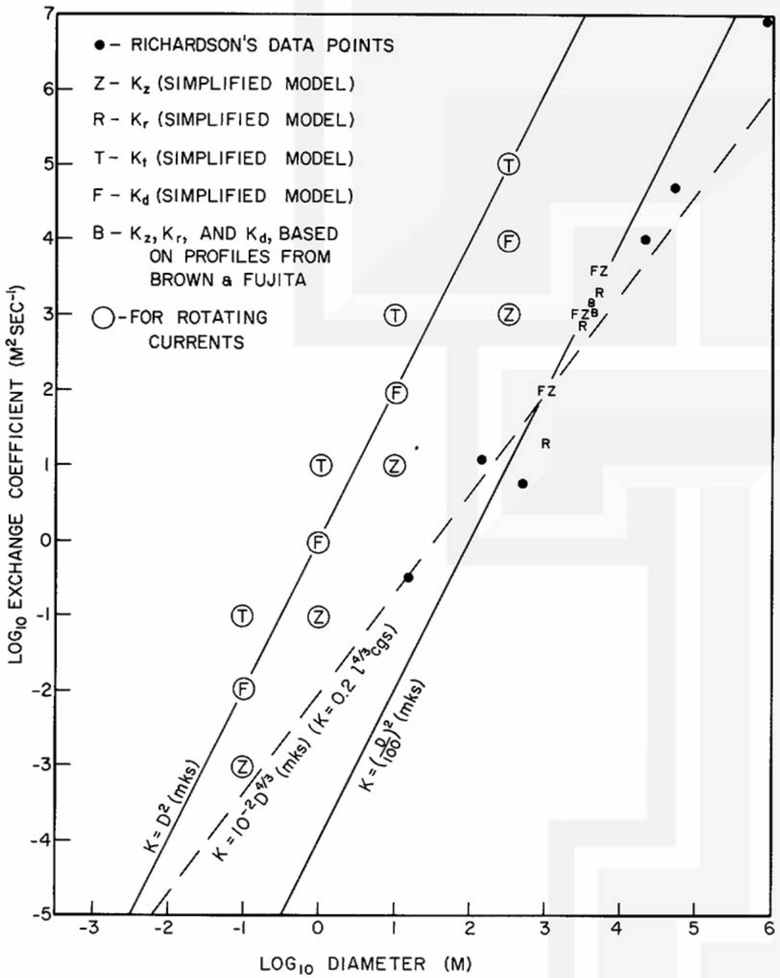

Fig. 11. Log-log plot of general eddy exchange coefficient (K) as a function of updraft diameter. Dashed line is based on the equation given by Richardson (1926) (Eq. (12)). Solid lines represent rotating and nonrotating updrafts.

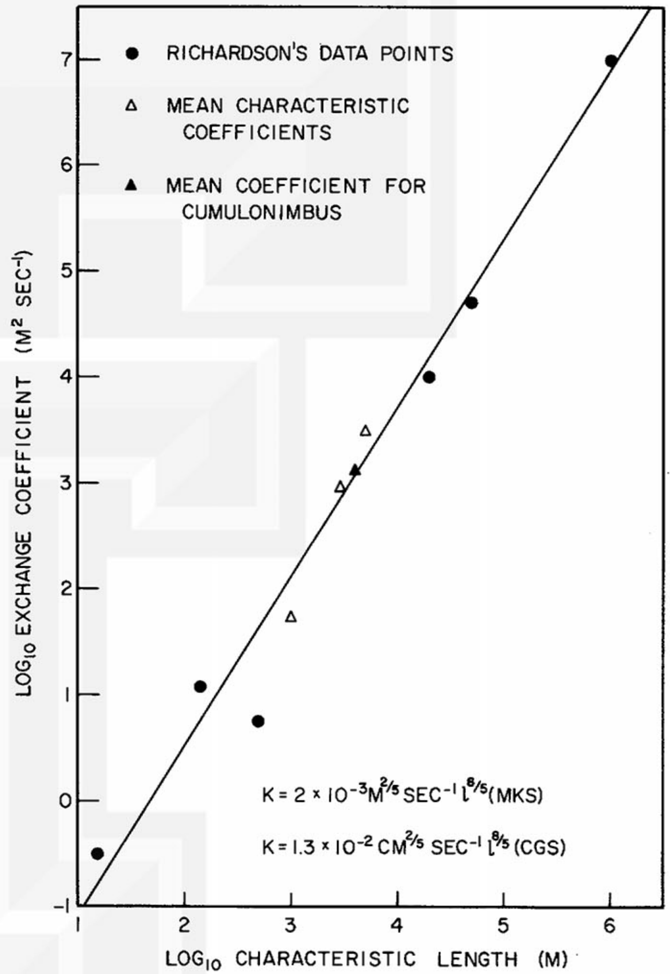

Fig. 12. Log-log plot of "universal" relation for general eddy exchange coefficient (K) as a function of a characteristic length (l) for nonrotating currents. Dots from Richardson (1926); open triangles from Table 3; solid triangles from Table 5.

7. Summary and Conclusion

The purpose of this paper has been twofold: first, to use a set of proposed equations to determine the order of magnitude of exchange coefficients in various-sized vortices and clouds and, second, to use empirical data to confirm that the proposed equations are physically realistic.

It has been shown that there is a marked difference between the range and order of magnitude of exchange coefficients for rotating and nonrotating convective currents. For a given diameter, the coefficients within a vortex are several orders of magnitude larger than those in nonrotating clouds. In vortices there is a two order of magnitude range in the various coefficients, while the larger clouds exhibit less than half an order of magnitude variation. This latter fact has been exploited intuitively, and also out of necessity, by investigators who have assumed that the viscosity and diffusion coefficients are equal. Summaries of the computed values for model vortices, model clouds and a cumulonimbus are given in Tables 2, 3, and 5, respectively.

Some of the coefficients computed for the cumulonimbus in Section 5 were of the order of $10^4 m^2 sec^{-1}$; one might argue that this is unrealistically high, that the viscosity term is larger than the others in the equation of motion. Therefore, it would be wise to make an order of magnitude check of the vertical equation of motion for clouds

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{dp}{dz} - g + K_z \nabla^2 w,$$

where w is vertical velocity, ρ is density, p is pressure, g is acceleration due to gravity, K_z is eddy viscosity coefficient for vertical motion, and ∇^2 is the Laplacian operator. The first term on the right is of the order of 1 to 10 m sec^{-2} in a small cumulonimbus, the second term is of the order of 10 m sec^{-2} and the third term is as high as 1 to 10 m sec^{-2} only if K_z is equal to $10^5 m^2 sec^{-1}$, which is an order of magnitude higher than any actually found. So, the computed orders of 10^3 and $10^4 m^2 sec^{-1}$ for a small cumulonimbus are quite realistic.

The most convincing evidence that has been presented to establish the implicit physical validity of the set of proposed equations is shown in Fig. 12; the computed values for both the model clouds and the cumulonimbus agree extremely well with noncloud data points. The consistency of all of the data has made it possible to modify Richardson's (1926) "universal" relation, which expresses an exchange coefficient (K) for all scales of phenomena as a function of a characteristic length

(l). The new relation is

$$K = 1.3 \times 10^{-2} cm^{2/5} \, sec^{-1} \, l^{8/5} \text{ (cgs)}$$

or

$$K = 2 \times 10^{-3} m^{2/5} \, sec^{-1} \, l^{8/5} \text{ (mks).}$$

The set of equations ((2) - (5)) proposed to be used to compute the eddy viscosity and diffusion coefficients within convective currents has been found to have implicit physical validity. This conclusion is based on the facts 1) that the computed values for the kinematic eddy viscosity coefficient in a cumulonimbus had excellent agreement with independently determined values (see Table 4) and 2) that the values of the eddy viscosity and diffusion coefficients computed from the equations for the cumulonimbus and for model clouds fall in line with independently determined values for other scales of atmospheric phenomena.

Therefore, it can be stated with considerable reliability that the set of equations presented in Section 2 will provide accurate values for the various exchange coefficients during numerical simulations of atmospheric processes. If only the proper order of magnitude of the coefficients is desired for a given sized convective current, the "universal" relation given above can be employed.

Acknowledgments

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REFERENCES

Brown, R.A., 1965: On the determination of the exchange coefficients in convective clouds. SMRP Research Paper No. 41, Univ. of Chicago, Dept. of Geophys. Sci., 7pp.

_____, and T. Fujita, 1965: A study of factors contributing to dissipation of energy in a developing cumulonimbus. SMRP Research Paper No. 42, Univ. of Chicago, Dept. of Geophys. Sci., 65 pp.

Byers, H.R., and R.R. Braham, 1948: Thunderstorm structure and circulation. J. Meteor, 5, 71-86.

Chou, H.-P., 1962: K voprosu o razvitii kuchevykh oblakov. [Problem of cumulus cloud development.] Izvestiia Akad. Nauk SSSR, Ser. Geofiz., No. 4, 548-557.

Christians, H., 1935: Zur dynamik der cumuluswolke. [The dynamics of cumulus clouds.] Beitr. Physik Fr. Atmos., 22, 149-160.

Goldman, J.L., 1965: Private communication.

Gutman, L.N., 1961: K teorii kuchevoĭ oblachnosti. [Theory of cumulus cloudiness.] Izvestiia Akad. Nauk SSSR, Ser. Geofiz., No. 7, 1040-1057.

Hoecker, W.H., 1960: Wind speed and air flow patterns in the Dallas tornado of April 2, 1957. Mon. Wea. Rev., 88, 167-180.

Lilly, D.K., 1962: On the numerical simulation of buoyant convection. Tellus, 14, 148-172.

_____, 1964: Numerical solutions for the shape-preserving two-dimensional thermal convection element. J. Atmos. Sci., 21, 83-98.

Malkus, J.S., and G. Witt, 1959: The evolution of a convective element: A numerical calculation. The Atmosphere and the Sea in Motion, New York, Rockefeller Inst. Press, 425-439.

Ogura, Y., 1963: The evolution of a moist convective element in a shallow, conditionally unstable atmosphere: A numerical calculation. J. Atmos. Sci., 20, 407-424.

Pinus, N.Z., 1963: Vertikal'nyye dvizheniya v grozovykh oblakakh. [Vertical movements in thunder clouds.] Doklady Akad. Nauk SSSR, 150, No. 4 788-790.

Richardson, L.F., 1926: Atmospheric diffusion shown on a distance-neighbour graph. Proc. R. Soc., London, Ser. A, 110, 709-737.

Schmidt, F.H., 1947: Some speculations on the resistance to the motion of cumuliform clouds. Meded. Verh., Ser. B, 1, No. 8, de Bilt, K. Ned. Meteor. Inst., 55 pp.

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