THEORY FOR THE DRIFT OF SEVERE LOCAL STORMS WITH APPLICATION TO THE STORM OF APRIL 21, 1961, NEAR TOPEKA, KANSAS

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1. INTRODUCTION

For over a decade observers have recognized that the most devasting tornadoes are spawned by thunderstorms that rotate (tornado cyclones) and that these storms tend to move at angles of 20° to 60° to the right of the mean wind. (See Browning and Fujita (1965) and Marwitz (1972).) In order to forecast tornadoes more accurately, the mechanism for this rightward movement must be understood. Understanding this mechanism should also contribute to our knowledge of severe storm structure, and this increased knowledge may point toward methods for positive satellite identification of severe local storms.

Theoretical approaches to this problem are quite diverse, although they fit generally into two categories: (a) those that are concerned primarily with convection and its propagation, such as Newton and Newton (1958), and Newton and Fankhauser (1964); and (b) those that are primarily concerned with the vortex and how it moves under the action of the Magnus force, the inertial force, and other forces, such as Goldman (1966), Fujita and Grandoso (1968), Costen (1970), and Goldman (1971). In both approaches the tornado cyclone is usually treated as a cylinder to conform with the known blocking effect of the updraft on the environmental winds.

This presentation fits into category (b) above. Its main theoretical feature is that the cylinder is recognized as buoyant and as tilted from the vertical so that the buoyant force has a component transverse to the axis of the cylinder. The rightward movement of tornado cyclones is postulated to be the steady drift required for the Magnus force generated by the drift to balance the transverse component of the buoyant force. This mechanism for the rightward movement was first proposed by Costen (1972).

Fujita suggested that the tilted tornado cyclone of April 21, 1961, as reported by Fujita and Arnold (1963), would be an excellent case for testing this theory. In a subsequent report, Fujita (1973b) further analyzed the data from this storm and presented it in a form convenient for application of the theory.

A feature of the tornado cyclone of April 21, 1961, is that its cyclonic circulation was found to be a decreasing function of altitude. The theoretical development to follow allows most quantities, including the circulation about the cylinder, to vary with altitude in order to conform with this data.

2. SYMBOL LIST

a	radius of buoyant circular cylinder that represents the core of a tornado cyclone, m
c	subscript that denotes axis of cylinder
C_1, C_2, D, E, G	integrals defined by equations (3)
g	acceleration of gravity, m/s^2
H	altitude of top of storm above ground, m
M'	mass per unit length of environmental air displaced by the circular cylinder, kg/m
ΔM	mass defect per unit length of the buoyant cylinder relative to the environment at the same altitude, kg/m
o	superscript or subscript that denotes the environment of the storm
p	pressure, mb
T	temperature, K
ΔT	$T_c - T_o$, temperature increment in the cylinder relative to the environment at the same altitude, K
$\vec{v}$	fluid velocity, m/s
(x,y,z)	local tangential Cartesian coordinate frame, where the x-axis points in the direction toward which the cylinder is tilted (Fig. 2), m
$\alpha, \beta, \gamma, \epsilon$	constants in equations (4)
Γ	cyclonic circulation, m^2/s

θ	tilt angle of cylinder from vertical, degrees
ρ	density, kg/m^3
$\Delta\rho$	$\rho_o - \rho_c$, density defect in the buoyant cylinder relative to the environment at the same altitude, kg/m^3
τ	period of trochoid resulting from superposition of inertial oscillation upon steady movement of tornado cyclone, s
(ξ,η,ζ)	tilted Cartesian frame shown in Figure 2, m
$\cdot$	dots above symbols denote time derivatives
$\langle\ \rangle$	bra-kets denote time-averaged values

3. THEORY FOR THE MOVEMENT OF TORNADO CYCLONES

3.1 Tornado Cyclone Model

According to the data presented by Fujita (1973b), the circulation Γ of the tornado cyclone of April 21, 1961, was much greater in magnitude and extent at lower altitudes than at mid- and high altitudes. Vortex lines which correspond qualitatively with this data are shown in Figure 1. The core of the tilted cyclone contains a sloping updraft that is fed from the low level flow and exhausts into the anvil flow near the tropopause. The updraft blocks the environmental flow and for this reason it has frequently been idealized as a solid cylinder. We shall incorporate this idealization into our tornado cyclone model.

Figure 1. Idealized vortex lines of a tilted tornado cyclone.

The model consists of a tilted, buoyant, circular cylinder with circulation $\Gamma(z)$ that decreases with altitude as vortex lines diverge into the environemntal flow, as shown in Figure 2. The environmental flow is essentially horizontal and varies with altitude; the horizontal vortex lines that correspond with the vertical shear of the environmental flow are not shown in Figures 1 and 2; also omitted are the circumferential vortex lines that surround the updraft column.

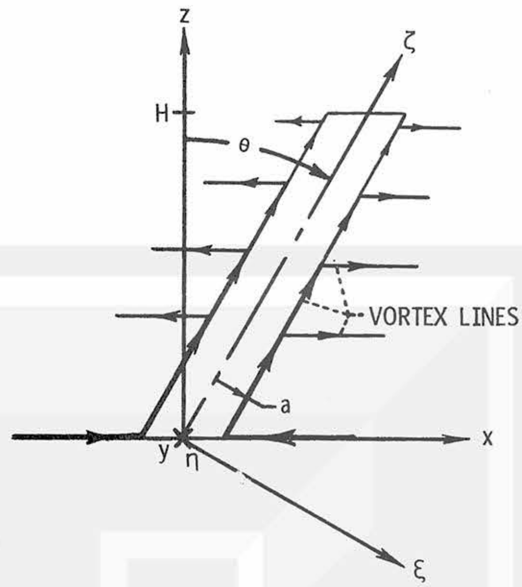

Figure 2. Model which represents the tornado cyclone of Figure 1 as a tilted, buoyant, circular cylinder with cyclonic circulation that decreases with altitude.

4.2 Equation for the Motion of a Tornado Cyclone

Figure 2 also shows the Cartesian coordinate frame (ξ,η,ζ) which is obtained from the local tangential frame (x,y,z) by a rotation about the y-axis through the tilt angle θ. The y- and η-axes are coincident, and the ζ-axis lies along the axis of the cylinder.

The motion of a buoyant circular cylinder with circulation was treated by Greenhill, and his solution is presented by Lamb (1945). The equations of momentum balance per unit length of the cylinder are given in the (ξ,η,ζ) frame by

$$\ddot{\xi}_c(2M' - \Delta M) + (\dot{\eta}_c - v_\eta^o)\Gamma\rho_o + g\Delta M \sin\theta = 0$$

$$\ddot{\eta}_c(2M' - \Delta M) - (\dot{\xi}_c - v_\xi^o)\Gamma\rho_o = 0 \qquad (1)$$

where (ξ_c,η_c) are the coordinates of the axis of the cylinder, and where the environmental fluid is considered to be inviscid and incompressible, although with variable density. The coefficient $(2M' - \Delta M)$ in the inertial terms represents the sum of the mass per unit length of the cylinder and of the environmental fluid that it displaces. The total balance of momentum on the cylinder is obtained by integrating equation (1) over the length of the cylinder (from the ground to the top of the storm). Transforming the integrated equations to the (x,y,z) coordinates then gives

$$\ddot{x}_c \cos\theta(2E - D) + \dot{y}_c G - C_2 + Dg\sin\theta = 0$$

$$\ddot{y}_c(2E - D) - \dot{x}_c G\cos\theta + C_1\cos\theta = 0 \qquad (2)$$

where

$$C_1 = \int_o^H v_x^o\,\Gamma\rho_o\,dz \qquad (3a)$$

$$C_2 = \int_0^H v_y^o \Gamma \rho_o \, dz \quad (3b)$$

$$D = \frac{\pi a^2}{\cos \theta} \int_0^H \Delta\rho \, dz \quad (3c)$$

$$E = \frac{\pi a^2}{\cos \theta} \int_0^H \rho_o \, dz \quad (3d)$$

$$G = \int_0^H \Gamma \rho_o \, dz \quad (3e)$$

In equations (3c) and (3d), $\pi a^2/\cos \theta$ is the area of a horizontal slice through the tilted cylinder. The density defect $\Delta\rho$ within the cylinder is idealized as being constant on this area, although it varies with altitude z.

Solutions of equations (2) for the motion of the tornado cyclone over the ground are given by

$$x_c = \alpha + t \frac{C_1}{G} + \frac{\gamma}{\cos \theta} \cos\left(\frac{Gt}{2E - D} + \epsilon\right)$$

$$y_c = \beta + t \frac{C_2 - Dg \sin \theta}{G} + \gamma \sin\left(\frac{Gt}{2E - D} + \epsilon\right) \quad (4)$$

where α, β, γ, and ϵ are constants. The path is a trochoid, where excursions in the x-direction are elongated by the factor $1/\cos \theta$. The trochoidal period is given by

$$\tau = \frac{2\pi(2E - D)}{G} \quad (5)$$

The oscillations that cause the movement to be trochoidal are generated by the inertial terms in equations (2) and shall be termed "inertial oscillations."

The x-component of the average displacement velocity is given by

$$\left\langle \dot{x}_c \right\rangle = \frac{C_1}{G} \quad (6a)$$

Therefore the average motion of the storm in the direction toward which it is tilted is given by the mean wind component weighted by $\rho_o \Gamma$. The y-component of average motion is given by

$$\left\langle \dot{y}_c \right\rangle = \frac{C_2 - Dg \sin \theta}{G} \quad (6b)$$

Thus the motion at right angles to the direction of tilt is the sum of the mean wind component weighted by $\rho_o \Gamma$ and the drift required for the Magnus force to balance the transverse component of the buoyant force on the cylinder. For a tornado cyclone tilted toward the east, this drift is southward - to the right of the mean wind.

4. APPLICATION TO THE TORNADO CYCLONE OF APRIL 21, 1961

The tornado cyclone of April 21, 1961, near Topeka, Kansas, is an exceptionally well-documented storm for checking equations (5) and (6) for the movement of tornado cyclones. This storm was probed by three Weather Bureau aircraft during the operational period of the National Severe Storms Project. Wind measurements at 914.4 m altitude were obtained by C. W. Newton aboard a B-26, and wind measurements at 6096 m and photographs showing the tilt of the storm were obtained by T. T. Fujita aboard a DC-6. Wind data obtained by a B-57 at 13.7 km were found to be misleading due to an equipment malfunction. As mentioned earlier, this documentation is presented by Fujita and Arnold (1963) and by Fujita (1973b). In the latter report, the data are analyzed and presented in a form convenient for computing the quantities C_1, C_2, D, E, and G in equations (3), (5), and (6).

The storm was tilted at an angle of 31° toward due east; therefore

$$\theta = 31^\circ \quad (7)$$

and the x-axis of coordinate frame (x,y,z) shown in Figure 2 points due east. The storm movement during its mature stage after 1700 CST was also due east at 11.33 m/s (22 kt), and the mature stage is the period for which the computation will be made. Therefore, the measured displacement velocity of the storm is given by

$$(\dot{x}_c, \dot{y}_c) = (11.33,\ 0) \text{ m/s} \quad (8)$$

Properties of the environment were determined by Fujita (1973b) from multi-level analyses of the wind and temperature fields at 1800 CST. The environmental wind components are presented in Table I and in the hodograph of Figure 3. The environmental pressure p_o and temperature T_o as functions of the altitude z are presented in Tables I and II. With these data, the environmental density ρ_o is determined from the equation

$$\rho_o = \frac{p_o}{2.87\ T_o} \quad (9)$$

and these values are also listed in Tables I and II.

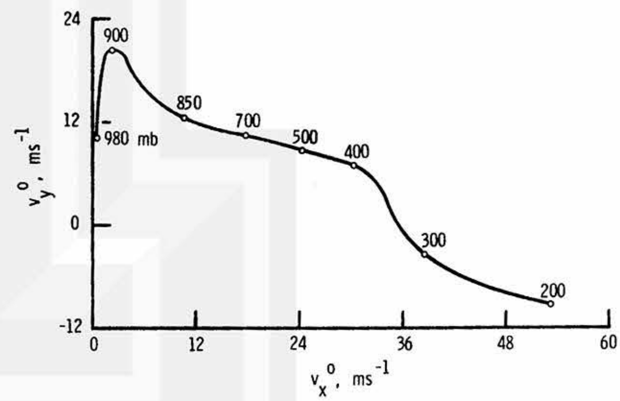

Figure 3. Hodograph of the environmental winds about the tornado cyclone of April 21, 1961, near Topeka, Kansas.

The circulation Γ of the tornado cyclone was calculated by Fujita from flight measurements made at 914.4 m and at 6096 m. At both of these altitudes, Γ was calculated from the measured velocity on circles of increasing radii that are centered on the strong echo region. According to these calculations, the cyclonic vorticity is spread out to at least 80 km radius

TABLE I. - CIRCULATION OF THE TORNADO CYCLONE OF APRIL 21. 1961, AND PROPERTIES OF ITS ATMOSPHERIC ENVIRONMENT.

z	p_0	v_x^0	v_y^0	ρ_0	Γ
m	mb	m/s	m/s	kg/m^3	m^2/s
0	980	0.5	10.5	1.150	1.91×10^6
1000	900	2.3	20.5	1.074	1.89
1556	850	10.6	12.6	1.026	1.72
3125	700	17.8	10.3	.877	1.23
5860	500	24.2	8.8	.667	.39
7500	400	30.3	7.0	.561	.25
9500	300	38.5	-3.4	.453	.14
12140	200	53.2	-9.4	.321	0

TABLE II. - ATMOSPHERIC PROPERTIES WITHIN AND NEAR THE TORNADO CYCLONE OF APRIL 21, 1961.

z	p_0	T_0	ΔT	ρ_0	$\Delta \rho$
m	mb	K	K	kg/m^3	kg/m^3
0	980	296.7	0	1.150	0
1×10^3	900	292.0	-2.0	1.074	$-.74 \times 10^{-2}$
2	810	286.0	-.3	.987	-.10
3	710	278.9	1.9	.887	.60
4	630	272.7	2.9	.805	.86
5	560	266.3	4.0	.733	1.10
6	490	260.2	4.3	.656	1.08
7	430	252.6	5.6	.593	1.32
8	370	244.0	6.0	.528	1.30
9	320	235.3	6.7	.474	1.35
10	280	225.8	7.2	.432	1.38
11	240	219.0	4.8	.382	.84
12	205	217.3	-2.9	.329	-.44
12.14	200	217.3	-4.5	.320	-.66

at 914.1 m altitude, and out to about 40 km radius at 6096 m altitude. The circulation values at these altitudes are

$$\Gamma(z) = \begin{cases} 1.91 \times 10^6 \ \mathrm{m^2/s} & (z = 914.4 \ \mathrm{m}) \\ 3.19 \times 10^5 \ \mathrm{m^2/s} & (z = 6096 \ \mathrm{m}) \end{cases} \quad (10)$$

These two points are connected by a straight line in Figure 4, where Γ (and also $\rho_o\Gamma$) are plotted against z. For $0 \leq z \leq 914.4$ m, Γ is taken to be constant because, within the earth's boundary layer, it seems unlikely that Γ could continue to increase as $z \rightarrow 0$. For $z > 6096$ m, the rapid decrease in Γ is slowed so that Γ extends to the top of the storm ($z = 12.14$ km) before decaying to zero. This behavior agrees with the notion that the convection and circulation are coupled, although it could not be checked by the flight data at 13.7 km, as explained earlier.

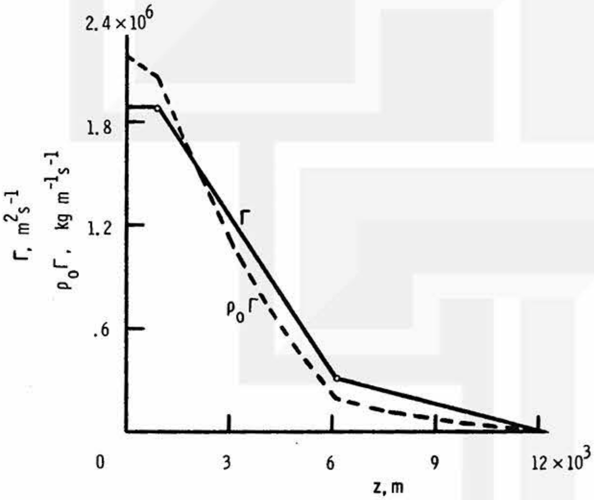

Figure 4. Plots of Γ and $\rho_o\Gamma$ against z for the tornado cyclone of April 21, 1961.

Values of $\Gamma(z)$ are also listed in Table I, and plots of $v_x^o\Gamma\rho_o$ and $v_y^o\Gamma\rho_o$ against z are shown in Figure 5. From these tables and plots, we determine three of the quantities (3)

$$C_1 = 9.25 \times 10^{10} \ \mathrm{kg\ m/s^2}$$
$$C_2 = 9.42 \times 10^{10} \ \mathrm{kg\ m/s^2}$$
$$G = 7.85 \times 10^9 \ \mathrm{kg/s} \quad (11)$$

and by equation (6a) the theoretical value for the easterly component of the storm's average movement is given by

$$\left\langle \dot{x}_c \right\rangle = 11.78 \ \mathrm{m/s} \quad (12)$$

Comparison with the measured value (8) shows a disagreement of only 4 percent and this is accountable by uncertainties in the data. Therefore, equation (6a) for the average movement of a tornado cyclone in the direction toward which it is tilted is supported by data from the tornado cyclone of April 21, 1961.

Result (12) was independent of the radius and buoyancy of the cylinder. In order to calculate the northward component of the storm's movement and the trochoidal frequency, we must determine the quantities D and E of set (3), which depend upon the radius and buoyancy. None of the three aircraft penetrated the core of the tornado cyclone; therefore, the radius a of the cylinder and the density defect $\Delta\rho$ within it were not directly measured.

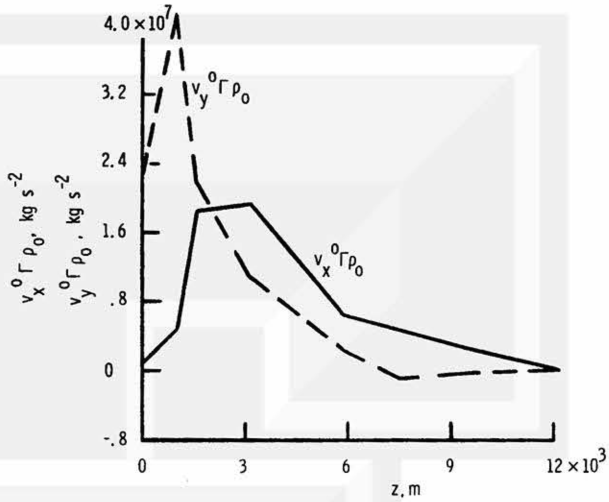

Figure 5. Plots of $v_x^o\Gamma\rho_o$ and $v_y^o\Gamma\rho_o$ against z for the tornado cyclone of April 21, 1961.

An estimate of the radius a of the cylinder can be obtained from the area of the strong radar echo at 1700 CST. At this time, although reaching maturity, the storm was still building, and during the building stage, the buoyant updraft core and the strong echo should approximately coincide, according to a private communication from P. C. Sinclair. The area of the strong echo at 1700 CST was 2.03×10^8 m^2, whence

$$\frac{\pi a^2}{\cos 31^\circ} = 2.03 \times 10^8 \ \mathrm{m^2} \quad (13)$$

and

$$a = 7442 \ \mathrm{m} \quad (14)$$

This value for a is consistent with updraft and temperature measurements made during other severe local storm penetrations by Sinclair (1973).

In order to estimate $\Delta\rho$, Fujita (1973b) calculated the temperature increment ΔT between the updraft and the environment as a function of z. These calculated values are listed in Table II. Underlying this estimate is the assumption of moist adiabatic ascent, without entrainment, of inflow air initially at temperature 297 K, dew point temperature 291 K, and station pressure 900 mb.

The small density defect $\Delta\rho$ between the environmental fluid and the updraft fluid at a given altitude is then determined from the perturbation equation

$$\Delta\rho = \frac{\rho_o}{T_o} \Delta T \quad (15)$$

which is a consequence of equation (9) and of the definitions for $\Delta\rho$ and ΔT.

Values of $\Delta\rho$ and ρ_o as functions of z are listed in Table II and are plotted in Figure 6. These values, together with result (13), give, when inserted into equations (3c) and (3d)

$$D = 1.76 \times 10^{10} \text{ kg}$$

$$E = 1.69 \times 10^{12} \text{ kg} \qquad (16)$$

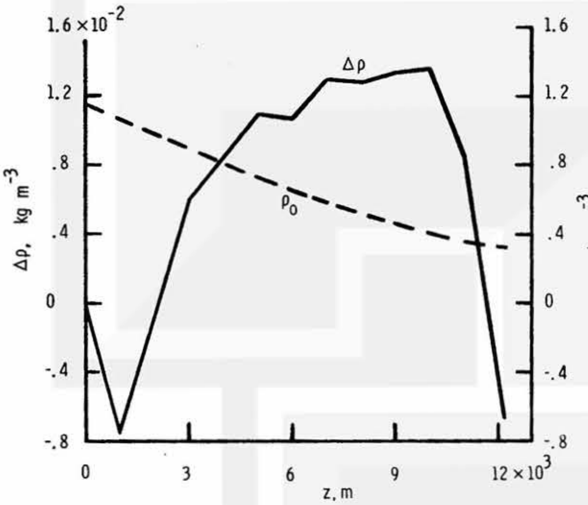

Figure 6. Plots of ρ_o and $\Delta\rho$ against z for the tornado cyclone of April 21, 1961.

Substituting equations (11) and (16) into equation (6b) gives, for the average northerly component of tornado cyclone movement

$$\left\langle \dot{y}_c \right\rangle = 0.665 \text{ m/s} \qquad (17)$$

The measured value (8) for this component is zero. However, the calculated value (17) is the difference of two nearly equal numbers that are an order of magnitude larger. Without the drift due to buoyancy, the calculation would have the storm moving northward at 12 m/s. Thus the calculated value (17) is actually in close agreement with the measured value for the northerly component of movement; and the model of a tilted, buoyant cylinder with circulation is supported by the data from the tornado cyclone of April 21, 1961.

The trochoidal period τ may be calculated from equations (5), (11), and (16)

$$\tau = 2691 \text{ s} \quad (45 \text{ minutes}) \qquad (18)$$

Although the subject tornado cyclone did not generate any tornadoes and no inertial oscillations were observed, the period of 45 minutes has been cited by Fujita (1973a) as characterizing the sequential generation of a family of tornadoes by a single tornado cyclone. Small amplitude inertial oscillations would alternately move the tornado cyclone toward and then away from the southerly inflow and tend to cause periodic variations in the updraft. Therefore, the trochoidal period of a tornado cyclone may be linked to the growth and decay of its tornadoes.

The short trochoidal period (18) precludes the possibility of attributing the rightward movement of the storm to its being on the rightward-moving half of the trochoid. On the trochoid, the storm could move to the right for at most 22.5 minutes, and it would then start moving to the left for the next 22.5 minutes. The tornado cyclone of April 21, 1961, was observed to move continuously to the right of the mean wind for over 2.5 hours. Therefore, its rightward movement must be due to the steady buoyant drift, as explained earlier.

5. CONCLUSIONS

The idealization of a tornado cyclone as a tilted, buoyant, circular cylinder with circulation is supported by data from the tornado cyclone of April 21, 1961, near Topeka, Kansas. According to this model, the tornado cyclone has trochoidal movement over the ground consisting of three additive components as follows: (1) the weighted mean wind, where the weighting factor is $\rho_o\Gamma$, the environmental density times the cyclonic circulation; (2) the buoyant drift (at right angles from the direction toward which the storm is tilted) such that the Magnus force generated by this drift balances the transverse component of the buoyant force on the tilted core; and (3) the inertial oscillation with period of about 45 minutes and arbitrary amplitude.

The characteristic movement of tornado cyclones to the right of the mean wind is attributed to the buoyant drift. The sequential generation of a family of tornadoes may be triggered by a small amplitude inertial oscillation of the tornado cyclone.

While the data from the storm of April 21, 1961, support this model, many more test cases need to be studied. Also, the connection between convective propagation and vortex movement needs to be investigated, since these two phenomena are evidently coupled in tornado cyclones.

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