SATELLITE & MESOMETEOROLOGY RESEARCH PROJECT

Department of the Geophysical Sciences
The University of Chicago

ON THE DETERMINATION OF THE EXCHANGE COEFFICIENTS IN CONVECTIVE CLOUDS

by

Rodger A. Brown

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ABSTRACT

The conventional micrometeorological equations for the evaluation of the exchange coefficients near the ground are not adequate when one attempts to apply them to a convective cloud. Therefore a set of equations is proposed which is based only on the wind field; these expressions are especially useful in determining the exchange coefficients in numerical simulations. Using a simplified cloud model, the orders of magnitude of the eddy viscosity coefficient for vertical motion in cumulus humilis, cumulus congestus, and cumulonimbus are found to be 10^2, 10^3, and 10^4 m^2 sec^{-1}, respectively.

1. Introduction

In the past few years, investigators in the field of cloud dynamics have been able to make use of electronic computers to solve the otherwise unmanageable non-linear differential equations. In the course of solving the equations that contain the coefficients of eddy diffusion and viscosity, it has been necessary to approximate these coefficients in one way or another. In most cases (Chou, 1962; Lilly, 1962, 1964; Ogura, 1963; and others), exchange coefficients are assumed to be constant during the entire simulation of a growing cloud. However, in some of his numerical experiments, Lilly (1962, 1964) did let the coefficients vary as a function of the grid-scale velocity and the grid-point separation.

It is the purpose of this note to present a set of equations that can be used to determine the exchange coefficients from the velocity data that are available at every grid point in a numerical simulation. The eddy viscosity coefficient for vertical motion will then be calculated for a range of simplified cloud models in order to test its validity.

2. Discussion of Equations

In the field of micrometeorology, the exchange coefficients are conventionally defined as (e. g., Priestley, 1959)

$$K_M = \frac{\tau}{\rho \frac{\partial \bar{u}}{\partial z}} , \tag{1}$$

$$K_H = - \frac{H}{\rho c_p \left(\frac{\partial \bar{T}}{\partial z} + \Gamma \right)} , \tag{2}$$

and

$$K_w = - \frac{E}{\rho \frac{\partial \bar{q}}{\partial z}} , \tag{3}$$

where K_M is the kinematic eddy viscosity coefficient, K_H is the eddy heat diffusion coefficient, K_w is the eddy water-vapor diffusion coefficient, τ is the stress, H is the heat flux, E is the water-vapor flux, Γ is the adiabatic lapse rate, and $\bar{T}$, $\bar{q}$ and $\bar{u}$ are the time-averaged values of the temperature, specific humidity, and horizontal component of the wind, respectively. One will note that only horizontal velocities and vertical gradients are taken into consideration.

Since the variables on the right side of the three equations are not all readily available, the writer decided to investigate variables that would be more amenable to measurement, especially in a numerical experiment. A simple dimensional analysis reveals that an exchange coefficient has the units of energy per unit mass divided by wind shear. This fact was taken into account in the development of the equations presented below.

In general, as the wind velocity in a cloud increases, the turbulence increases and consequently one would expect that there would be greater diffusion and viscosity acting in such a situation. Therefore, as a continuation of this idea, it will be implicit in this discussion that the exchange processes are the direct result of turbulence and that <u>all</u> of the exchange coefficients are solely a function of the velocity distribution.

The type of energy that is most important in this situation is the kinetic energy of the flow. The pertinent shear is the shear normal to the direction of motion. As seen in (1), the conventional eddy viscosity coefficient is for horizontal motion only. Here three viscosity coefficients are being proposed, one for each of three mutually-orthogonal directions. Likewise, micrometeorologists use different diffusion coefficients for heat and water vapor; both will be expressed by the same equation in the proposed scheme. Therefore, the following set of equations are proposed for computing the exchange coefficients in convective clouds:

$$K_r = \frac{\frac{1}{2} v_r^2}{\left| \frac{\partial v_r}{\partial z} \right|} , \qquad (4)$$

$$K_\theta = \frac{\frac{1}{2} v_\theta^2}{\left| \frac{\partial v_\theta}{\partial z} \right|} , \qquad (5)$$

$$K_z = \frac{\frac{1}{2} w^2}{\left| \frac{\partial w}{\partial r} \right|} , \qquad (6)$$

and

$$K_d = \frac{\frac{1}{2} (v_r^2 + v_\theta^2 + w^2)}{\left| \frac{\partial}{\partial z} (v_r + v_\theta) \right| + \left| \frac{\partial w}{\partial r} \right|} , \qquad (7)$$

where K_r , K_θ , and K_z are the eddy viscosity coefficients for radial (v_r), tangential (v_θ), and vertical (w) velocities, respectively, and where K_d is the eddy diffusion coefficient for both heat and moisture.

It is assumed that the exchange coefficients are positive, as indicated by the absolute magnitude bars in the denominator of each equation. For the set of equations (4) - (7), axial symmetry has been assumed; in the case of a growing stationary cloud, it is easiest to use axial symmetry in the numerical simulation.

3. The Eddy Viscosity Coefficient for Vertical Motion

In order to test the accuracy of relative order of magnitude for this set of equations, it is necessary to compare the values computed from actual measurements in a cloud. The author knows of only two attempts to approximate any of the coefficients; in both instances only the eddy viscosity coefficient for vertical motion (K_z) was sought. Richardson (1921) calculated a value of $10^2 m^2 sec^{-1}$ for small cumulus clouds having a mean vertical velocity of $6.5 \times 10^{-2} m \, sec^{-1}$. Based on aircraft acceleration measurements, Ogura (1963) estimated that the order of magnitude of K_z in a thunderstorm is roughly $10^3 m^2 sec^{-1}$.

There have been many investigators (Christians, 1935; Schmidt, 1947; Gutman, 1961; Chou, 1962; Lilly, 1962; Ogura, 1963; and others) who have indicated their preference for the order of magnitude of the eddy viscosity coefficient in cumulus clouds. Their estimates ranged from 1 to several times $10^2 m^2 sec^{-1}$, with $10^2 m^2 sec^{-1}$ being most prevalent. It is interesting to note that none of the authors cited above made any reference to the computations of Richardson (1921); yet they deduced the same order of magnitude.

In connection with his numerical simulations of moist convective elements (i. e., simple cumulus clouds), Ogura (1963) was able to get an idea of the lower limit of K_z (in actuality he used the same value for all of the exchange coefficients). For three different runs, keeping all other parameters constant, he let K_z vary from 40 to 4 to 0 $m^2 sec^{-1}$, respectively. There was no appreciable difference in the evolution of the convective elements for 4 and 0 $m^2 sec^{-1}$. This would indicate that the order of 10 $m^2 sec^{-1}$ is the lowest significant value one could expect in a cumulus cloud.

We shall now calculate K_z for a variety of in-cloud conditions. The following simplified model will be used to evaluate K_z in which: 1) the vertical velocity in the numerator of (6) is equal to the horizontal mean, 2) the horizontal mean (across the cloud) of the vertical velocity is one-half the maximum vertical velocity at the center of the cloud at that level, and 3) the representative shear at that level is equal to the maximum velocity divided by the radius of the cloud. Using this model for the updraft stage of a cloud, (6) reduces to

$$K_z = \frac{1}{8} \overline{w} D , \qquad (8)$$

where $\overline{w}$ is the horizontal mean of the vertical velocity and D is the diameter of the cloud at the same level.

The results of evaluating (8) are shown in Fig. 1, where the values of K_z are plotted as a function of $\overline{w}$ and D. One of the most obvious conclusions to be drawn is that 10 $m^2 sec^{-1}$ is, in fact, a lower limit of the values which can be expected; it corresponds to a cloud a few hundred meters in diameter having a mean vertical velocity of less than 0.5 m sec^{-1}. Richardson's value of $10^2 m^2 sec^{-1}$ does not seem to apply to the curves in the figure; perhaps this is due to the fact his vertical velocity was the average for an 8.5-hr period.

Itemized in Table 1 are some of the characteristic parameters for cumulus humilis, cumulus congestus, and cumulonimbus clouds. The column on the extreme right gives an approximate mean value of the eddy viscosity coefficient for vertical motion. The range in the order of magnitude may be more than one would expect. However, it should be remembered that the investigators cited at the beginning of this section were mainly concerned with smaller cumulus clouds, where the coefficient should be about $10^2 m^2 sec^{-1}$. Except for Ogura's rough estimate, no one has expressed any interest in cumulonimbi and, consequently, the order of $10^4 m^2 sec^{-1}$ is not mentioned in the literature investigated here.

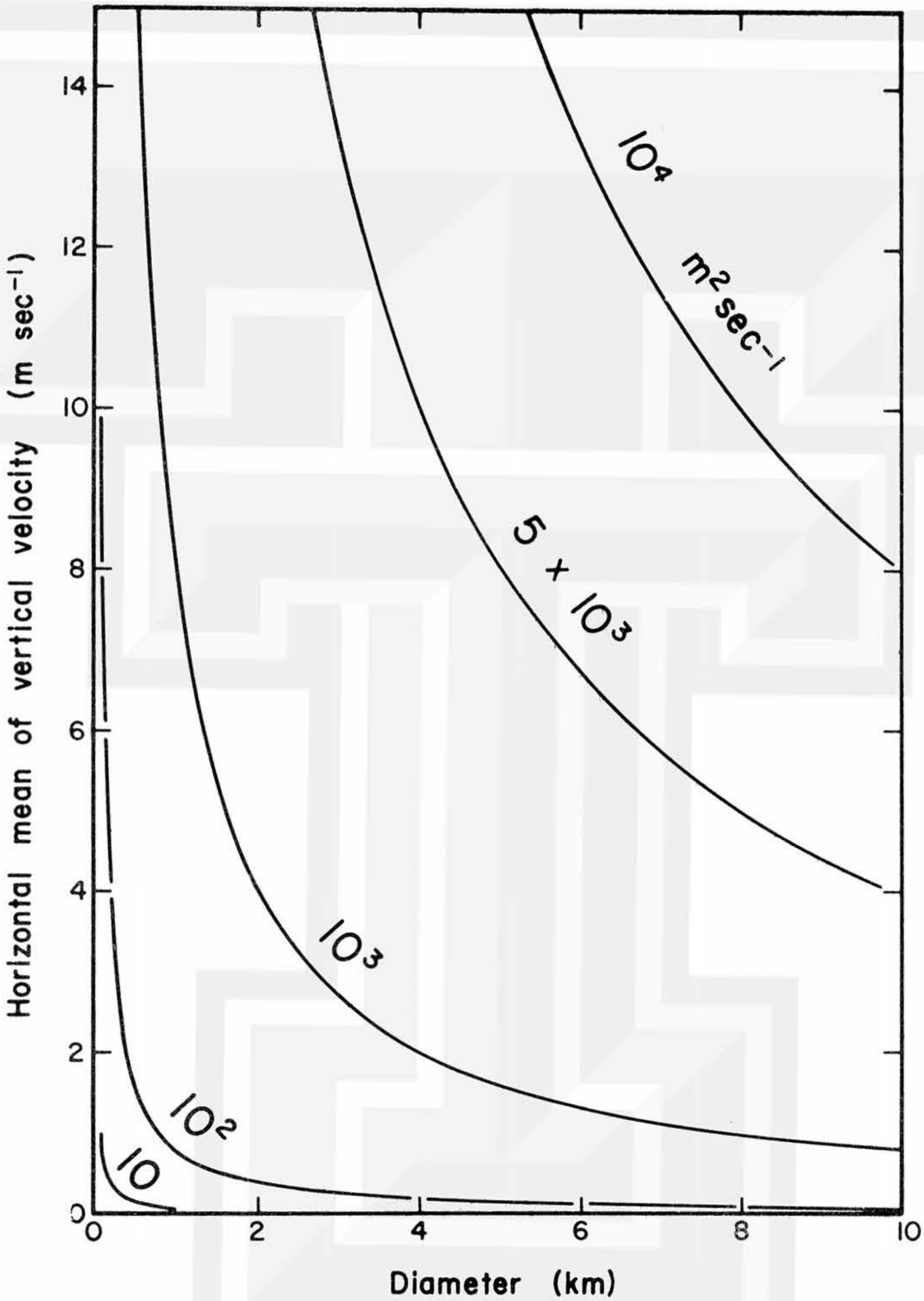

Fig. 1. The range of values of the eddy viscosity coefficient for vertical motion that can be expected in clouds having the indicated diameters and mean vertical velocities.

Table 1. Characteristic values of cloud diameter (D), mean vertical velocity ($\overline{w}$) and eddy viscosity coefficient (K_z) for the three types of clouds.

Type	D (km)	$\overline{w}$ (m sec^{-1})	K_z ($m^2 sec^{-1}$)	$\overline{K_z}$ ($m^2 sec^{-1}$)
Cu hu	1	≤ 3	$\leq 4 \times 10^2$	10^2
Cu con	3	≤ 7	$\leq 2.5 \times 10^3$	10^3
Cb	10	≤ 15	$\leq 2 \times 10^4$	10^4

The data presented in Fig. 1 help to substantiate the physical validity of the proposed set of equations. Smaller clouds, with little energy available, have lower eddy viscosity coefficients. As clouds become larger and have more energy available for growth, the coefficient increases proportionately due to increased turbulence. If small clouds had a coefficient of $10^4 m^2 sec^{-1}$, they could never develop. Likewise, if cumulonimbi had a coefficient of 10^2 they would be able to grow practically without bounds, which is not the observed case. Therefore, the figure (and the equation upon which it is based) suggests a natural coupling between the amount of energy available for cloud growth and the amount of energy dissipation due to the resulting turbulence.

4. Conclusion

In this note, a proposed set of equations for the exchange coefficients in a convective cloud has been presented. It is not being advocated that these equations should replace the ones now being used by micrometeorologists. However, it is strongly recommended that they should be evaluated in those situations where Eqs. (1) - (3) apply in order to see how they compare; it has been arbitrarily assumed here that the apparent success of K_z also applies to the other exchange coefficients.

As mentioned in the introduction, this study was prompted by a need for a consistent, yet realistic, set of equations that could be used to determine the exchange coefficients in numerical simulations of clouds. This becomes especially important when one attempts to simulate the growth of small cumulus clouds into a thunderstorm, a process involving at least two orders of magnitude change in the exchange coefficients. Whenever such a simulation is eventually undertaken, it is needless to say that ingenuity will have to be employed in order to minimize computational instability and other non-meteorological problems.

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